WIT

BACHELOR OF SCIENCE (HONS) IN
- APPLIED COMPUTING
- COMPUTER FORENSICS & SECURITY
- ENTERTAINMENT SYSTEMS

EXAMINATION:

DISCRETE MATHEMATICS (COMMON MODULE)

SEMESTER 1 - YEAR 1

DECEMBER 2013 **DURATION: 2 HOURS**

INTERNAL EXAMINER:	**MS ANNE DALY WALSH**	**DATE:**	**16 DECEMBER, 2013.**
		TIME:	**16.45 PM**
		VENUE:	**MAIN HALL**
EXTERNAL EXAMINERS:	**DR FLAITHRÍ NEFF**		
	DR ANTHONY KEANE		
	PROF M-TAHAR KECHADI		

INSTRUCTIONS TO CANDIDATES

1. **ANSWER *THREE* QUESTIONS.**
2. **TOTAL MARK IS 150.**
3. **MARKS MAY BE LOST IF ALL WORK IS NOT SHOWN CLEARLY**

MATERIALS REQUIRED

1. **NEW MATHEMATICS TABLES.**

WATERFORD INSTITUTE OF TECHNOLOGY

Question 1

(a) The universal set is {1, 2, 3, 4, 5, 6, ………….., 18}.

Let P = {2, 3, 5, 12, 15, 18} and Q = {2, 5, 7, 8, 9, 10, 12} and

R = {1, 2, 3, 4, 5, 7, 11}.

(i) Represent the sets **U**, P, Q and R in a Venn diagram, marking all the elements in the appropriate places.

(5 marks)

(ii) List the elements of the following sets:

(1) $P^c \cap Q,$

(2) $P \oplus R$

(3) $R \backslash (P \cup Q)^c$

(10 marks)

(b)

(i) Given $\{(x, y) | \ x, y \in A,$ and $x^2 - y^2$ is divisible by 3} when A is the set {0, 1, 2, 3}

(1) Express the relation as a set of ordered pairs.

(2) Draw a digraph to represent this relation.

(3) Investigate whether the relation is an equivalence relation? Explain your answer fully.

(10 marks)

(ii) $f: Z \to Z, f(x) = -x + 5$ and $g: Z \to Z, g(x) = x^2 - 2x + 1$

(1) Find $f \circ g$ and $g \circ f$.

(2) Explain why f has an inverse but g does not.

(10 marks)

Question 1 continued overleaf

Question 1 continued

(c)

(i) Investigate using Venn diagrams for all sets X, Y and Z whether

$$X\backslash(Y \cup Z) \Leftrightarrow (X\backslash Y) \cup Z$$

(5 marks)

(ii) Prove using 'is an element of' and the laws of set theory that

$$(A \cap B)^{\,c} \cap A \Leftrightarrow (A^c \cup B^c) \cap A$$

(5 marks)

(iii) Prove using set identities that

$$[A \cup (B \cap C)]^c \Leftrightarrow (A^c \cap C^c) \cup (A^c \cap \ B^c)$$

(5 marks)

(Total 50 Marks)

Question 2

(a)

(i) Construct the truth table for:

$$[\overline{(p \times q)} + (r \times \bar{q})]$$

or

$$[\neg(p \ \wedge \ q) \vee (r \wedge \neg\, q)]$$

(7 marks)

(ii) Using a truth table prove that the following logical equivalence is a tautology.

$$[(p + q).(p \rightarrow r).(q \rightarrow r)] \rightarrow r$$

or

$$[(p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r)] \rightarrow r$$

(8 marks)

Question 2 continued overleaf

(b)

(1) Using the laws of logic (at the end of the exam paper), investigate if

$$\overline{\left(p + (\bar{p}.q)\right)} \Leftrightarrow \bar{p}.\bar{q}$$

or

$$\neg\left(p \vee (\neg p \wedge q)\right) \Leftrightarrow \neg p \wedge \neg q$$

Ensure you specify which of the logic laws you are using at each stage of the proof.

(6 marks)

(2) Simplify fully the following Boolean expression:

$$[\bar{q}\ (p \rightarrow q)] \rightarrow \bar{p}$$

or

$$[\neg\, q \wedge (p \rightarrow q)] \rightarrow \neg\, p$$

(9 marks)

(c)

(i) Let $P = \{2, 3, 4, 5, 7, 8, 9, 10, 11, 12, 15\}$.

Determine the truth value of each of the following statements for $x \in P$. Explain your answer fully in each case.

(1) If x is odd or $x > 11$ then $x < 15$.

(2) If x is odd then $\exists x \epsilon P,\ x < 7$.

(3) If $(x < 7) \vee (x > 10)$ then x is odd.

(4) If $(x > 4) \wedge (x < 5)$ then x is the null set.

(8 marks)

(ii) Prove by mathematical induction

$$\frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \frac{1}{4.5} + \cdots + \frac{1}{n(n+1)} = \frac{n}{n+1}, \forall n \in N_0,$$

(12 marks)

(Total 50 Marks)

Question 3

(a)

(i) How many different ways are there of arranging the letters in the word

DISCONNECTED

(2 marks)

(ii) How many of these ways start with the consecutive string "DIS"?

(2 marks)

(iii) How many even numbers are in the range 1000 and 9999 if there are no repeating digits?

(4 marks)

(iv) How many committees of six people can be chosen from 10 men and 15 women?

(1) If exactly four women must be on each committee.

(2) If at least four women must be on each committee.

(7 marks)

(b) Consider the binomial expansion of $(3x^3 - 2y^2)^8$

(1) Write out the first three terms and simplify fully.

(2) Evaluate the coefficient of y^{10}?

(15 marks)

(c)

(i) Construct a generating function G(x) for the number of ways of selecting 2 red, 5 green and 3 blue balls from a pack if at least 1 red ball and at least two green balls must be selected.

(10 marks)

(ii) Determine using generating functions the number of ways of selecting 6 balls from the pack above.

(10 marks)

(Total 50 Marks)

Question 4

(a)

(i) Draw the graph with five vertices of the following degrees:

3, 3, 4, 4, 4

(3 marks)

(ii) Draw the following graphs $K_{2,3}$ and W_3

(4 marks)

(iii) Explain fully what is meant if a graph is bipartite using the graph in **Fig1** below

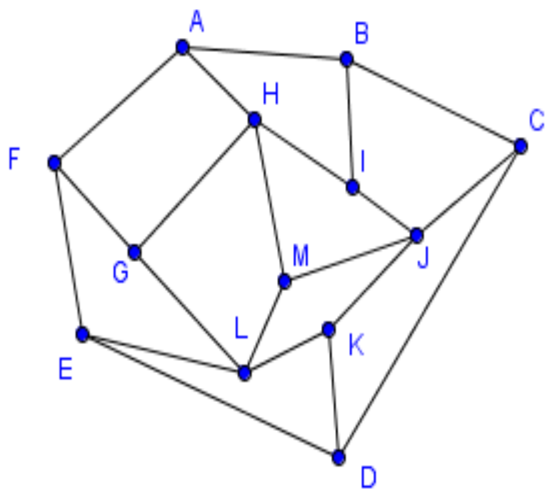

Fig 1

(8 marks)

Question 4 continued overleaf

(b)

(i) Investigate the graph in **Fig 2** below to see whether it has a Eulerian circuit. Construct such a circuit if one exists. If it does not exist give an argument to show why no such circuit exists.

(ii) Show the graph in **Fig 2** below is Hamiltonian. Confirm with an appropriate diagram.

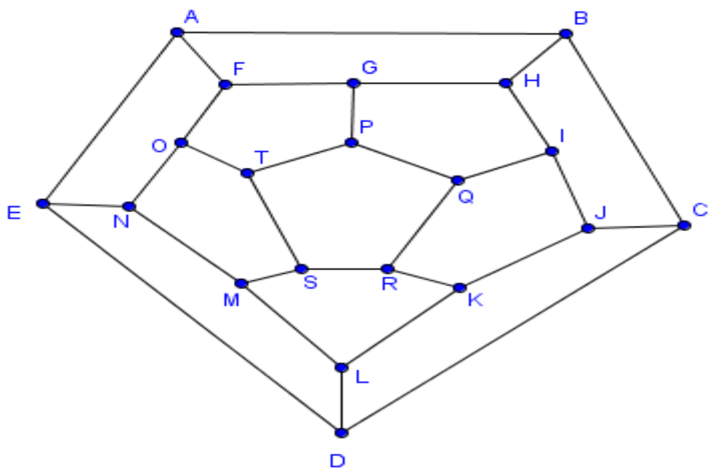

Fig 2

(8 marks)

(iii) Write out the degree of each node A to G and investigate whether the graph in **Fig 3** below is Eulerian. Justify your answer.

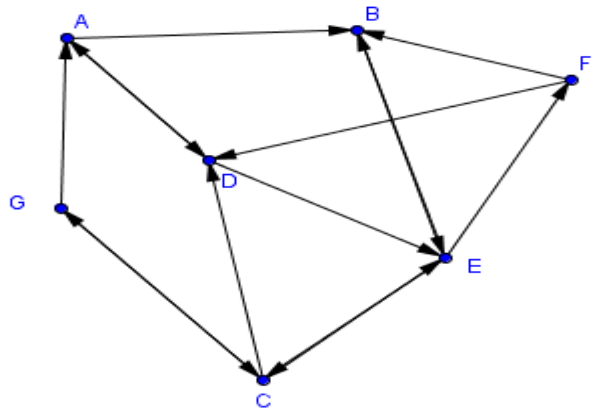

Fig 3

(8 marks)

Question 4 continued overleaf

(iii) Using **Fig 3** above write out the corresponding adjacency matrix.

(4 marks)

(c)

(i) A sequence is defined recursively by $a_0 = 1$, $a_1 = 4$ and

$a_n = 4a_{n-1} - 4a_{n-2}$ for $n \geq 2$. Find the first 5 terms of the sequence.

(3 marks)

(ii) Find $\sum_{n=1}^{\infty} \frac{1}{(n+2)(n+3)}$ using the method of partial fractions.

(12 marks)

(Total 50 Marks)

PLEASE ASK FOR THE NEW MATHEMATICS TABLES

Laws of Set Theory

Equivalence	***Name***
$A \cap U \Leftrightarrow A$ $A \cup \emptyset \Leftrightarrow A$	Identity Laws
$A \cap \emptyset \Leftrightarrow \emptyset$ $A \cup U \Leftrightarrow U$	Domination Laws/Bound Laws
$A \cap A \Leftrightarrow A$ $A \cup A \Leftrightarrow A$	Idempotent Laws
$(A^c)^c \Leftrightarrow A$ or $(A')' \Leftrightarrow A$	Double Negation
$A \cap B \Leftrightarrow B \cap A$ $A \cup B \Leftrightarrow B \cup A$	Commutative Laws
$(A \cap B) \cap C \Leftrightarrow A \cap (B \cap C)$ $(A \cup B) \cup C \Leftrightarrow A \cup (B \cup C)$	Associative Laws
$A \cup (B \cap C) \Leftrightarrow (A \cup B) \cap (A \cup C)$ $A \cap (B \cup C) \Leftrightarrow (A \cap B) \cup (A \cap C)$	Distributive Laws
$(A \cap B)^c \Leftrightarrow A^c \cup B^c$ or $(A \cap B)' \Leftrightarrow A' \cup B'$ $(A \cup B)^c \Leftrightarrow A^c \cap B^c$ or $(A \cup B)' \Leftrightarrow A' \cap B'$	De Morgan's Law
$A \cup A^c \Leftrightarrow U$ $A \cap A^c \Leftrightarrow \emptyset$	Complement Laws
$\emptyset^c \Leftrightarrow U$ $U^c \Leftrightarrow \emptyset$	0/1 Laws
$A \cap (A \cup B) \Leftrightarrow A$ $A \cup (A \cap B) \Leftrightarrow A$	Absorption laws

Table of Logical Equivalences Boolean Algebra symbols

Equivalence	***Name***
$p \cdot 1 \Leftrightarrow p$ $p + 0 \Leftrightarrow p$	Identity Laws
$p + 1 \Leftrightarrow 1$ $p \cdot 0 \Leftrightarrow 0$	Domination Laws
$p + p \Leftrightarrow p$ $p \cdot p \Leftrightarrow p$	Idempotent Laws
$\overline{(\overline{p})} \Leftrightarrow p$	Double Negation
$p + q \Leftrightarrow q + p$ $p \cdot q \Leftrightarrow q \cdot p$	Commutative Laws
$(p + q) + r \Leftrightarrow p + (q + r)$ $(p \cdot q) \cdot r \Leftrightarrow p \cdot (q \cdot r)$	Associative Laws
$p + (q \cdot r) \Leftrightarrow (p + q) \cdot (p + r)$ $p \cdot (q + r) \Leftrightarrow (p \cdot q) + (p \cdot r)$	Distributive Laws
$\overline{(p \cdot q)} \Leftrightarrow \overline{p} + \overline{q}$ $\overline{(p + q)} \Leftrightarrow \overline{p} \cdot \overline{q}$	De Morgan's Law
$p + \overline{p} \Leftrightarrow 1$ $p \cdot \overline{p} \Leftrightarrow 0$	Complement Laws
$p \rightarrow q \Leftrightarrow \overline{p} + q \Leftrightarrow \overline{q} \Rightarrow \overline{p} \Leftrightarrow \overline{p} \Rightarrow \overline{q}$	Material Implication (MI)
$p(p + q) \Leftrightarrow p$ $p + pq \Leftrightarrow p$	Absorption Laws

Table of Logical Equivalences Discrete Maths symbols

Equivalence	***Name***
$p \wedge T \Leftrightarrow p$ $p \vee F \Leftrightarrow p$	Identity Laws
$p\vee T \Leftrightarrow T$ $p \wedge F \Leftrightarrow F$	Domination Laws
$p\vee p \Leftrightarrow p$ $p \wedge p \Leftrightarrow p$	Idempotent Laws
$\neg (\neg p) \Leftrightarrow p$	Double Negation
$p \vee q \Leftrightarrow q \vee p$ $p \wedge q \Leftrightarrow q \wedge p$	Commutative Laws
$(p\vee q) \vee r \Leftrightarrow p \vee (q \vee r)$ $(p \wedge q) \wedge r \Leftrightarrow p \wedge (q \wedge r)$	Associative Laws
$p \vee (q \wedge r) \Leftrightarrow (p \vee q) \wedge (p \vee r)$ $p \wedge (q \vee r) \Leftrightarrow (p \wedge q) \vee (p \wedge r)$	Distributive Laws
$\neg(p \wedge q) \Leftrightarrow \neg p \vee \neg q$ $\neg(p \vee q) \Leftrightarrow \neg p \wedge \neg q$	De Morgan’s Law
$p \vee \neg p \Leftrightarrow T$ $p \wedge \neg p \Leftrightarrow F$	Complement Laws
$p \rightarrow q \Leftrightarrow \neg p \vee q$ $\Leftrightarrow \neg q \rightarrow \neg p \Leftrightarrow \neg p \rightarrow \neg q$	Material Implication (MI)