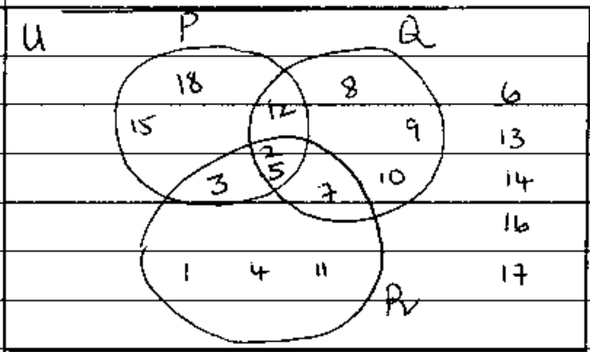
Semester I - Dec 2013

Question No.& Part	Subject: Discrete Maths Examiner: Ms. Anne Daly Walsh	Marks Total
1 (a)	$U = \{1, 2, 3, \cdots\cdots 18\}$	
	$P = \{2, 3, 5, 12, 15, 18\}$	
	$Q = \{2, 5, 7, 8, 9, 10, 12\}$	
	$R = \{1, 2, 3, 4, 5, 7, 11\}$	
(i)		5
(ii)	(1) $P^c \cap Q$	
	$P^c = \{1, 4, 6, 7, 8, 9, 10, 11, 13, 14, 16, 17\}$	
	$P^c \cap Q = \{7, 8, 9, 10\}$	3
	(2) $P \oplus R = (P \cup R) \setminus (P \cap R)$	
	$= \{1, 4, 7, 11, 12, 15, 18\}$	3
	(3) $R \setminus (P \cup Q)^c$	
	$P \cup Q = \{2, 3, 5, 7, 8, 9, 10, 12, 15, 18\}$	
	$(P \cup Q)^c = \{1, 4, 6, 11, 13, 14, 16, 17\}$	

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	$R \setminus (P \cup Q)^c$	
	$\{1,2,3,4,5,7,11\} \setminus \{1,4,6,11,13,14,16,17\}$	
	$= \{2,3,5,7\}$	4
(b)	$\{(x,y) \mid x,y \in A$ and x^2-y^2 is divisible by 3$\}$ when $A = \{0,1,2,3\}$	
	(1) $0^2-0^2=0$ $(0,0)$	
	$0^2-1^2=-1$	
	$0^2-2^2=-4$	
	$0^2-3^2=-9$ $(0,3)$	
	$1^2-0^2=1$	
	$1^2-1^2=0$ $(1,1)$	
	$1^2-2^2=3$ $(1,2)$	
	$1^2-3^2=$	
	$2^2-0^2=$	
	$2^2-1^2=3$ $(2,1)$	
	$2^2-2^2=0$ $(2,2)$	
	$2^2-3^2=$	
	$3^2-0^2=9$ $(3,0)$	
	$3^2-1^2=$	
	$3^2-2^2=$	
	$3^2-3^2=0$ $(3,3)$	

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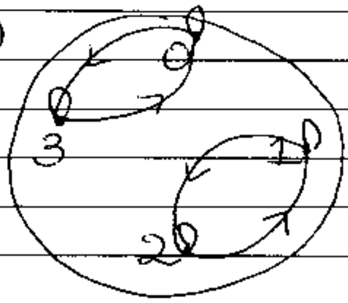
	R = {(0,0), (0,3), (1,1), (1,2), (2,1), (2,2) (3,0), (3,3)}	3
	(2) 	2
	(3) Equivalence Relation must be Reflexive, symmetric & transitive.	
	Reflexive: Yes (0,0) (1,1), (2,2), (3,3) As there is $(x,x) \in R$, $\forall x \in A$ is true	
	Symmetric: Yes (0,3), (3,0) (1,2), (2,1) as $\forall x,y \in A$, if $(x,y) \in R$ then $(y,x) \in R$ is true.	
	Transitive: Yes. $\forall x,y,z \in A$ if $(x,y) \in R$ & $(y,z) \in R$ then $(x,z) \in R$ is true	

(0,0) (0,3) ∴ (0,3)

⇒ equivalence relation. 5

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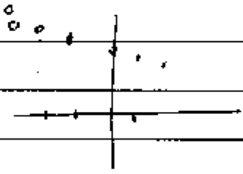
b (ii) (1)	$f: \mathbb{Z} \to \mathbb{Z},\ f(x) = -x+5$	
	$g: \mathbb{Z} \to \mathbb{Z},\ g(x) = x^2-2x+1$	
	$f \circ g = f(g(x))$	
	$f(x^2-2x+1) = -(x^2-2x+1)+5$	
	$= -x^2+2x+4$	2
	$g \circ f = g(f(x))$	
	$g(-x+5) = (-x+5)^2 - 2(-x+5)+1$	
	$= x^2-10x+25+2x-10+1$	
	$= x^2-8x+16$	2
(2)	f has an inverse as:	
	$-x+5 = y$ 	
	$x = -(y-5)$	
	$y = -3$	
	$x = -(-3-5) = 8$	
	$f(8) = -8+5 = -3$	
1–1	$f(a_1) \neq f(a_2)$ and	
onto	all $y \in \mathbb{Z}$ are used	
	g: has not got an inverse	3
	x^2-2x+1	

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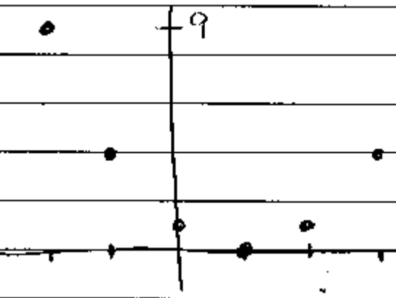
	$x = -2 \quad (-2)^2 - 2(-2) + 1 =$	
	$4 + 4 + 1 = 9$	
	$x = -1 \quad (-1)^2 - 2(-1) + 1 = 4$	
	$x = 0 \quad 0^2 - 2(0) + 1 = 1$	
	$x = 1 \quad 1^2 - 2(1) + 1 = 0$	
	$x = 2 \quad 2^2 - 2(2) + 1 = +1$	
	$x = 3 \quad 3^2 - 2(3 + 1 = 4$	
		
	No negative y-values used	
	∴ not onto	
	∴ g does not have an inverse	3

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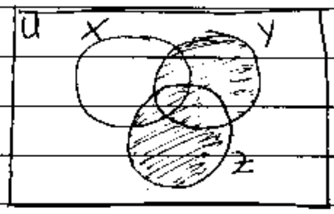
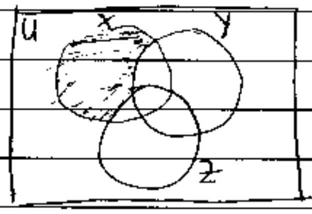
(c)(i)	$X \setminus (Y \cup Z) = (X \setminus Y) \cup Z$?	

LHS

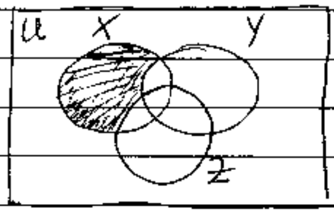


$Y \cup Z$

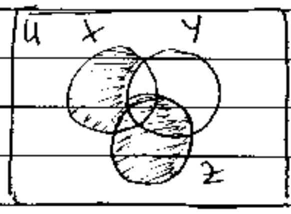


$X \setminus (Y \cup Z)$

RHS



$X \setminus Y$



$(X \setminus Y) \cup Z$

LHS ≠ RHS

$\therefore X \setminus (Y \cup Z) \neq (X \setminus Y) \cup Z$ — 5.

(ii) $(A \cap B)^C \cap A \Leftrightarrow (A^C \cup B^C) \cap A$

$x \in (A \cap B)^C \cap A \Leftrightarrow$

$\Leftrightarrow x \in (A \cap B)^C$ and $x \in A$

$\Leftrightarrow x \in (A^C \cup B^C)$ and $x \in A$

$\Leftrightarrow (x \in A^C$ OR $x \in B^C)$ and $x \in A$

$\Leftrightarrow x \in (A^C \cup B^C) \cap A$ — 5.

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(C) (iii)		
	$[A \cup (B \cap C)]^c = (A^c \cap \ ^c) \cup (A^c \cap B^c)$	
	$A^c \cap (B \cap C)^c$ de Morgans law	
	$A^c \cap (B^c \cup C^c)$ de Morgan's	
	$(A^c \cap B^c) \cup (A^c \cap C^c)$ Distributive law	
	$(A^c \cap C^c) \cup (A^c \cap B^c)$ Commutative Law	
		5

Q2. (a)

(i)

$\bar{q}$	p q r	p×q	A: $\overline{p \times q}$	B: $r \times \bar{q}$	A+B
1	0 0 0	0	1	0	1
1	0 0 1	0	1	1	1
0	0 1 0	0	1	0	1
0	0 1 1	0	1	0	1
1	1 0 0	0	1	0	1
1	1 0 1	0	1	1	1
0	1 1 0	1	0	0	0
0	1 1 1	1	0	0	0

7

(ii) $[(p+q)(p \to r).(q \to r)] \to r$

p	q	r	A: p+q	B: p→r	C: q→r	A.B	X: AB.C	X→r
0	0	0	0	1	1	0	0	1
0	0	1	0	1	1	0	0	1
0	1	0	1	1	0	1	0	1
0	1	1	1	1	1	1	1	1
1	0	0	1	0	1	0	0	1
1	0	1	1	1	1	1	1	1
1	1	0	1	0	0	0	0	1
1	1	1	1	1	1	1	1	1

This is a tautology as all entries in result are true

8

b) (i)

(1) $\overline{p + (\bar{p} \cdot q)} \Leftrightarrow \bar{p} \cdot \bar{q}$

$\bar{p} \cdot (\overline{\bar{p} \cdot q})$ De Morgans.

$\bar{p} \cdot (p + \bar{q})$ De Morgans.

$\bar{p} \cdot p + \bar{p}\,\bar{q}$ Distributive.

$0 + \bar{p} \cdot \bar{q}$ Complement

$\bar{p} \cdot \bar{q}$ Domination — 5.

(2) $[\bar{q}\,(p \Rightarrow q)] \rightarrow \bar{p}$

$[\bar{q}\,(\bar{p} + q)] \rightarrow \bar{p}$ Mat Implication

$(\bar{q} \cdot \bar{p} + \bar{q} \cdot q) \rightarrow \bar{p}$ Distributive

$(\bar{q}\,\bar{p} + 0) \rightarrow \bar{p}$ Complement

$\bar{q}\,\bar{p} \rightarrow \bar{p}$ Identity

$\overline{\bar{q}\,\bar{p}} + \bar{p}$ Mat Imp.

$q + p + \bar{p}$ de Morgans

$q + 1$ Complement — 9

1 Domination

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(c)	i $P = \{2,3,4,5,7,8,9,10,11,12,15\}$	
	(1) $x \in P$ is odd OR $x > 11$ then $x < 15$	
	$\{15\}$	
	false as $x = 15$	2
	(2) $x \in P$ is odd, then $\exists x \in P$, $x < 7$	
	$\{3,5,7,9,11,15\}$	
	true as $3, 5 < 7$.	
	So $\exists x \in P$, $x < 7$.	2
	(3) $(x<7) \cup (x > 10)$ then x is odd	
	$\{3,5\} \cup \{11,12,15\} = \{3,5,11,12,15\}$	
	false	
	as 12 is not odd	2
	(4) $\{(x>4) \cap (x<5)$ then x is the null set.	
	$\{5,7,8,9,10,11,12,15\} \cap$	
	$\{2,3,4\}$	
	$= \{\ \}$ ∴ true	2

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C(iii)	$\frac{1}{1\cdot2}+\frac{1}{2\cdot3}+\frac{1}{3\cdot4}+\dots+\frac{1}{n(n+1)}=\frac{n}{n+1}$	
	$n=1$	
	$\frac{1}{1\cdot2}=\frac{1}{2}$ $\quad \frac{1}{1+1}=\frac{1}{2}$	
	true for $n=1$	
	Assume true for $n=k$	
	$\frac{1}{1\cdot2}+\frac{1}{2\cdot3}+\frac{1}{3\cdot4}+\dots+\frac{1}{k(k+1)}=\frac{k}{k+1}$	
	Prove true for $n=k+1$	
	$\left(\frac{1}{1\cdot2}+\frac{1}{2\cdot3}+\frac{1}{3\cdot4}+\dots+\frac{1}{k(k+1)}\right)+\frac{1}{(k+1)(k+2)}=\frac{k+1}{k+2}$	
	↓	
	$\frac{k}{k+1}+\frac{1}{(k+1)(k+2)}$	
	$\frac{k(k+2)+1}{(k+1)(k+2)}$	
	$\frac{k^2+2k+1}{(k+1)(k+2)}=\frac{(k+1)(k+1)}{(k+1)(k+2)}$	
	$=\frac{k+1}{k+2}$	
	LHS = RHS	12
	$\frac{1}{1\cdot2}+\frac{1}{2\cdot3}+\frac{1}{3\cdot4}+\dots+\frac{1}{n(n+1)}=\frac{n}{n+1}$	
	$\forall n \in N_0$.	

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Q3 (a)	DISCONNECTED	
(i)	$\frac{12!}{2!\,2!\,2!\,2!} = 29,937,600$	2
(ii)	DIS unit $\frac{9!}{2!\,2!\,2!} = 45,360$	2
(iii)	1000 — 9999	
	9x8x7 [ ] Even 0,2,4,6,8 — 10 digits, 1 used; 0	
	9x8x7x1 = 504	
	8x8x7 [ ] 2,4,6,8 — 4; No zero here	4
	8x8x7x4 = 1792.	
	504 + 1792 = 2296	
(iv)	10 men 15 women (6)	
(1)	$^{15}C_4 \times {}^{10}C_2 = 1365 \times 45 = 61,425$	2
(2)	4 women & 2 men = 61,425	
	5 women & 1 man = $^{15}C_5 \times {}^{10}C_1$	
	6 women + 0 men = $^{15}C_6 \times {}^{10}C_0$	

	61,425 $\quad$ 61,425	
	$3003 \times 10 =$ $\quad$ 30,030	
	$5005 \times 1 =$ $\quad$ 5005	5
	96,460	
(b) (1)	$(3x^3 - 2y^2)^8$	
	$\binom{8}{0}(3x^3)^8(-2y^2)^0 + \binom{8}{1}(3x^3)^7(-2y^2)^1$	
	$+ \binom{8}{2}(3x^3)^6(-2y^2)^2 + \ldots$	5
	$6561x^{24} + (8)(2187x^{21})(-2y^2)$	
	$+ 28(729x^{18})(4y^4)$	
	$6561x^{24} - 34992x^{21}y^2$	
	$+ 81,648x^{18}y^4$	5
(2)	$\binom{8}{r-1}(3x^3)^{\cancel{10-(r-1)}}(-2y^2)^{r-1}$	
	y^{10}	
	$(-2y^2)^{r-1} = (-2)^{r-1}(y^2)^{r-1}$	
	$y^{2r-2} = y^{10}$ $\quad$ $\boxed{r=6}$	

OR assume $(y^2)^x = y^{10}$

$\boxed{x=5}$

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	$\binom{8}{5}(3x^3)^3(-2y^2)^5$	
	$\binom{8}{5}(3)^3(-2)^5 = -48384$	5.
(c)	6 balls	
	2 Red at least 1 $x + x^2$	
	5 green at least 2 $x^2 + x^3 + x^4 + x^5$	
	3 blue $1 + x + x^2 + x^3$	
	$f(x) = (x + x^2)(x^2 + x^3 + x^4 + x^5)(1 + x + x^2 + x^3)$	
	$f(x) = x^3(1+x)(1 + x + x^2 + x^3)(1 + x + x^2 + x^3)$	
	$f(x) = x^3\left(\frac{1-x^2}{1-x}\right)\left(\frac{1-x^4}{1-x}\right)^2$	
	$= \frac{x^3}{(1-x)^2}\,(1-x^2)(1-x^4)^2$	10.
	$(1-x)^{-2} = 1 + \binom{3}{1}x + \binom{4}{2}x^2 + \binom{5}{3}x^3 + \cdots$	
	$(1-x^4)^2 = 1 - 2x^4 + x^8$	
	$(1-x^2)(1-2x^4+x^8)$	
	$1(1-2x^4+x^8) - x^2(1-2x^4+x^8)$	
	$1 - 2x^4 + x^8 - x^2 + 2x^6 - x^{10}$	
	$1 - x^2 - 2x^4 + 2x^6 + x^8 - x^{10}$	

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$$f(x) = x^3\left(1 + \binom{3}{1}x + \binom{4}{2}x^2 + \binom{5}{3}x^3 + \cdots\right)$$

$$\left(1 - x^2 - 2x^4 + 2x^6 + x^8 - x^{10}\right)$$

6 balls.

$$x^3 \left(\quad\right)^3\left(\quad\right)^0 = \binom{5}{3}(1)$$

$$\left(\quad\right)^0\left(\quad\right)^3 = (1)(0)$$

$$\left(\quad\right)^2\left(\quad\right)^1 \quad \binom{4}{2}(0)$$

$$\left(\quad\right)^1\left(\quad\right)^2 \quad \binom{3}{1}(-1)$$

$$\binom{5}{3} - \binom{3}{1} = 10 - 3 = 7$$ 10

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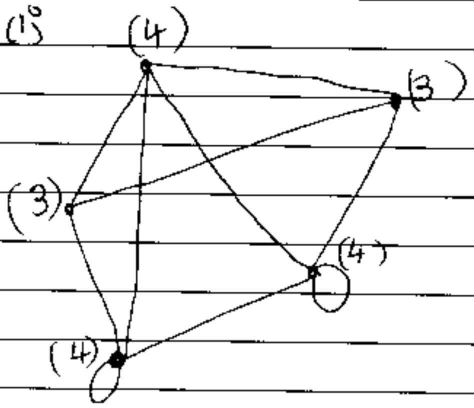
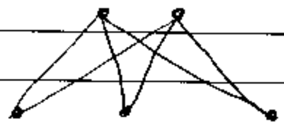
4 (a)	(i)	

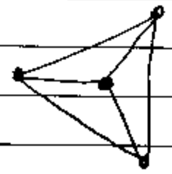


3.

(ii)



$K_{2,3}$ — 2



W_3 — 2

(iii)

A	(A,B), (A,H), (A,F)
C	(C,B) (C,J) (C,D)
E	(E,D), (E,F), (E,L)
G	(G,F), (G,H), (G,L)
I	(I,B), (I,H), (I,J)
K	(K,D), (K,J), (K,L)

M (M,J), (M,L) (M,H)

Domain = { A, C, E, G, I, K, M }
Range = { B, D, F, H, J, L }

A graph is bipartite if it can be written in such a way that the domain & Range have no common letters.

(b)(i)

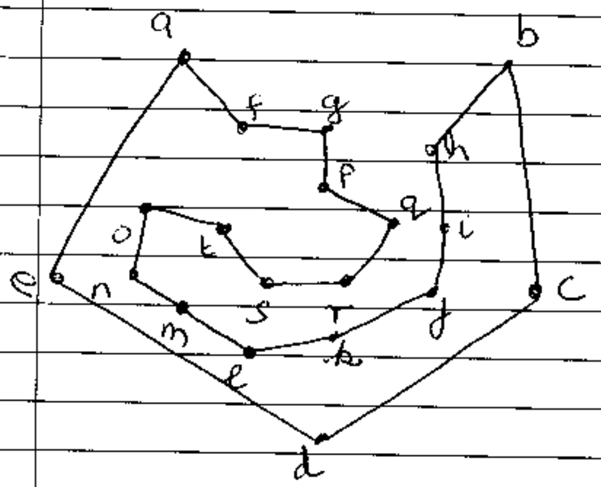
A = 3	L = 3
B = 3	M = 3
C = 3	N = 3
D = 3	O = 3
E = 3	P = 3
F = 3	Q = 3
G = 3	R = 3
H = 3	S = 3
I = 3	T = 3
J = 3	
K = 3	

3/

This is not a Eulerian circuit as all vertices are not even.

(ii) Hamiltonian circuit: all vertices must have degree 2 — & you must use all vertices in the graph & begins & ends at the same vertex.

2/



3

(iii) A

Vertex	IN	out
A	2	2 ✓
B	3	1
C	2	3
D	3	2
E	3	3 ✓
F	1	2
G	1	2

4

No: Eulerian circuit: same indeg as out degree 2

Eulerian path: 2 nodes – have a difference of 1 between indeg & out degree 2

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(iv)

	A	B	C	D	E	F	G
A	0	1	0	1	0	0	0
B	0	0	0	0	1	0	0
C	0	0	0	1	1	0	1
D	1	0	0	0	1	0	0
E	0	1	1	0	0	1	0
F	0	1	0	1	0	0	0
G	1	0	1	0	0	0	0

4

(C) (i) $a_0 = 1 \quad a_1 = 4$

$a_n = 4a_{n-1} - 4a_{n-2} \quad n \geq 2$

$a_2 = 4a_1 - 4a_0$

$= 4(4) - 4(1) = 12.$

$a_3 = 4a_2 - 4a_1$

$= 4(12) - 4(4) = 32.$

$a_4 = 4a_3 - 4a_2$

$= 4(32) - 4(12) = 80$

$1, 4, 12, 32, 80, \ldots\ldots$

3

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(ii)	2, 4, 8, 16	1
	$a = 2 \quad r = 2$	
	$T_n = (2)(2)^{n-1}$	
	$T_n = 2^n$ Explicit	2
	$a_0 = 2$	
	$a_1 = 4$	
	$a_1 = 2(2)$	
	$= 2a_0$	
	$a_2 = 2a_1$	
	$2(4) = 8$	2
	$\boxed{a_n = 2a_{n-1}}$	
(iii)	$\frac{1}{(n+1)(n+3)} = \frac{A}{n+1} + \frac{B}{n+3}$	
	$\frac{1}{(n+1)(n+3)} = \frac{A(n+3) + B(n+1)}{(n+1)(n+3)}$	
	$1 = A(n+3) + B(n+1)$	
	$n+1=0 \quad n=-1$ | $n+3=0 \quad n=-3$	
	$1 = A(2)$ | $1 = -2B$	
	$\frac{1}{2} = A$ | $-\frac{1}{2} = B$	4

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	$\frac{1}{(n+1)(n+3)} = \frac{\frac{1}{2}}{n+1} - \frac{\frac{1}{2}}{n+3}$	
	$n=1$ $\quad \frac{\frac{1}{2}}{2} - \frac{\frac{1}{2}}{4}$	
	$n=2$ $\quad \frac{\frac{1}{2}}{3} - \frac{\frac{1}{2}}{5}$	
	$n=3$ $\quad \frac{\frac{1}{2}}{4} - \frac{\frac{1}{2}}{6}$	
	$\vdots$ $\quad - \frac{\frac{1}{2}}{n+1}$	
	$n=n-2$	
	$n=n-1$ $\quad \frac{\frac{1}{2}}{n} - \frac{\frac{1}{2}}{n+2}$	
	$n=n$ $\quad \frac{\frac{1}{2}}{n+1} - \frac{\frac{1}{2}}{n+3}$	
	$S_n = \frac{\frac{1}{2}}{2} + \frac{\frac{1}{2}}{3} - \frac{\frac{1}{2}}{n+2} - \frac{\frac{1}{2}}{n+3}$	5
	$S_n = \frac{1}{4} + \frac{1}{6} - \frac{1}{2}\left[\frac{1}{n+2} + \frac{1}{n+3}\right]$	
	$S_\infty = \frac{5}{12}$	3