WIT

BACHELOR OF SCIENCE (HONS) IN
- APPLIED COMPUTING
- COMPUTER FORENSICS & SECURITY
- ENTERTAINMENT SYSTEMS

EXAMINATION:

DISCRETE MATHEMATICS (COMMON MODULE)

SEMESTER 1 - YEAR 1

DECEMBER 2014 **DURATION: 2 HOURS**

INTERNAL EXAMINER:	MS ANNE DALY WALSH	DATE:	15 DECEMBER, 2014
		TIME:	14.15 PM
		VENUE:	MAIN HALL
EXTERNAL EXAMINERS:	DR ANTHONY KEANE		
	PROF M-TAHAR KECHADI		
	DR ANDREW ERRITY		

INSTRUCTIONS TO CANDIDATES

1. ANSWER *THREE* QUESTIONS.
2. TOTAL MARK IS 150.
3. MARKS MAY BE LOST IF ALL WORK IS NOT SHOWN CLEARLY

MATERIALS REQUIRED

1. NEW MATHEMATICS TABLES.

WATERFORD INSTITUTE OF TECHNOLOGY

Question 1

(a) The universal set is {0, 1, 2, 3, 4, 5, 6, ………….,15}

Let A = {3, 4, 6, 9, 10, 14, 15} and B = {3, 4, 5, 8, 10, 11} and

C = {3, 5, 6, 7, 11, 12, 14}.

(i) Represent the sets **U**, A, B and C in a Venn diagram, marking all the elements in the appropriate places.

(5 marks)

(ii) List the elements of the following sets:
(Explain your answers fully by showing **ALL** intermediate steps)

(1) $(A \cup B)^C \backslash (A \cap B^c)$

(2) $C \backslash (A \cap B)^c$

(10 marks)

(b) **(i)** Given the relation R such that R = $\{(m, n) \in R \mid m, n \in A,$ and $m^2 - n \in A\}$ when A is the set {0, 1, 2, 3, 4}

(1) Express the relation R as a set of ordered pairs.

(2 marks)

(2) Draw a digraph to represent this relation.

(2 marks)

(3) Investigate whether the relation is antisymmetric and/or transitive.
(Explain your answer fully using the validity of recognised definitions **and** specifics from the example.

(6 marks)

(ii) Given $f(x) = -\sqrt{3x-6} + 12$ when $x \geq 2$. Find the domain of the function.
Hence find $f^{-1}(x)$ and give the domain for $f^{-1}(x)$.

(10 marks)

Question 1 (c) continued overleaf

(c) Investigate whether $(P \cap Q) \cup (P^C \cup R)^C$ is equivalent to $(P \cup Q) \cap (P \cup Q^C) \cap (P^c \cup Q \cup R^c)$, using a method of set theory. Specify which law is valid at each step.(See tables at end of paper)

(15 marks)

(Total 50 marks)

Question 2

(a) Construct a truth table that simplifies the following expression and investigate whether the result is a tautology, contradiction or neither.

$$[(p+q)\times(p\times r)] \rightarrow (r \leftrightarrow p)$$

(10 marks)

(b) Simplify fully using the laws of logic (at the end of the exam paper) the expression

$$\overline{[p+q\times r]} \rightarrow \overline{p}\times\overline{q}$$

[Ensure you specify which of the logic laws you are using at each stage of the proof]

(15 marks)

(c)

(1) Let $M = \{0, 2, 4, 6, \ldots\ldots, 20\}$.

List the elements of the appropriate solution set for each of the following statements for $x \in M$ and determine the truth value of each of the statements. **(Explain your answers fully by showing ALL intermediate steps)**

- **a.** If x is a multiple of 4 or $x > 11$ then, $\exists x > 12$.
- **b.** If x is a multiple of 3, then $x > 6, \forall x \in M$.
- **c.** $\forall x \in M, (x < 7) \vee (x > 8)$.
- **d.** $\exists x \in M, (x \geq 4) \wedge (x < 5)$.

(4 x 2.5 marks)

Question 2 continues overleaf

(2) Prove by mathematical induction

$$1^3 + 2^3 + 3^3 + \cdots . + n^3 = \frac{n^2(n+1)^2}{4}, \forall n \in N$$

where N = {1, 2, 3, 4, ……}.

(15 marks)

(Total 50 marks)

Question 3

(a) Expand and simplify fully $(2x^2 - y)^{10}$

(10 marks)

(b) **(i)** Use appropriate rules in probability to find how many numbers in the :

(1) Range of numbers from 500 to 9999 if there are no restrictions?

(2) Range of numbers from 500 to 9999 if the number must be odd.

(3) Range of numbers from 500 to 9999 if no digit can be repeated.
[Marks may be lost if method is not shown]

(3x5 marks)

(ii) From a haulage company's records, 4% of deliveries are made in error. If twenty trips are planned next week, calculate the probability that there will be at least two erroneous deliveries, using the binomial distribution method.

(10 marks)

(c) Construct generating functions for the number of ways of selecting 5 red, 6 green and 4 blue balls from a pack if at least 3 red ball and at least 4 green balls must be selected. Hence determine using generating functions the number of ways of selecting10 balls from the pack above.

(15 marks)

(Total 50 marks)

Question 4

(a)

(i) Draw the **simple** graph with six vertices of the following degrees:

2, 3, 3, 4, 5, 5

(5 marks)

(ii) Construct a $K_{4,3}$ graph (if possible) from the graph in **Fig1** below and investigate whether the graph is bipartite. Justify your answer using appropriate criteria for bipartite.

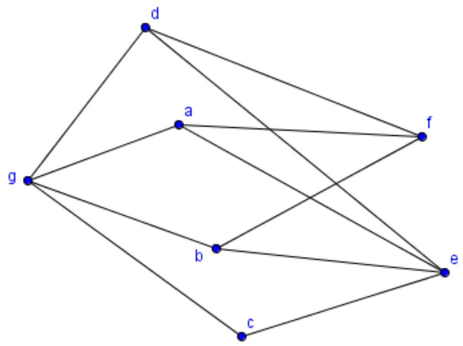


Fig 1

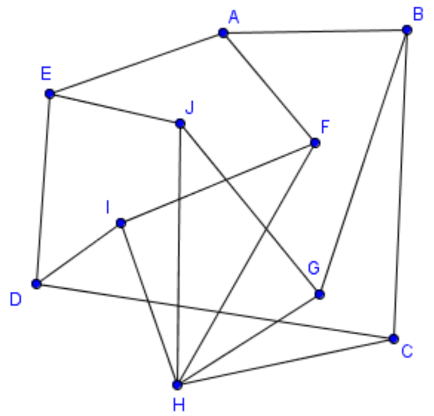
(10 marks)

Question 4(b) continues overleaf

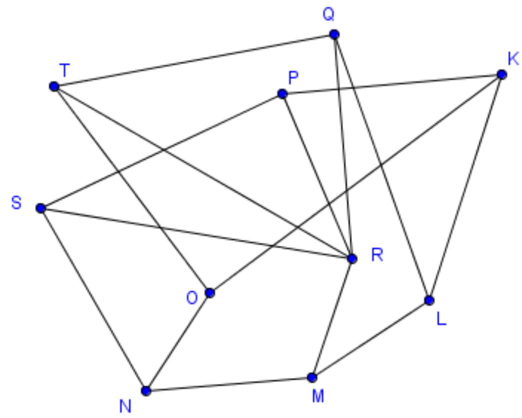
Question 4

(b)

(i) Investigate whether graphs G and H in **Fig 2** below are isomorphic. Justify your answer using appropriate criteria for isomorphism.



G



H

Fig 2

(10 marks)

(ii) Write out the degree of each node A to G and investigate whether the graph in **Fig 3** below is Eulerian. Justify your answer.

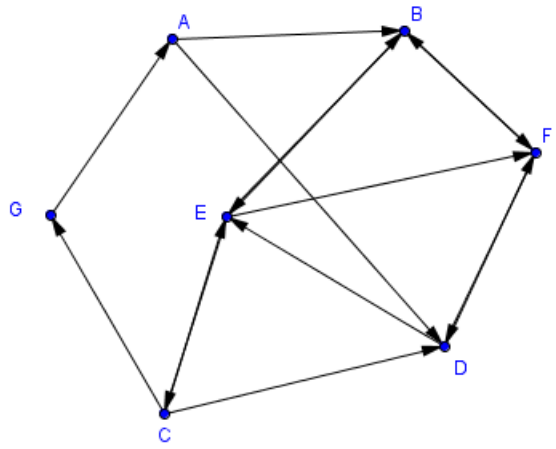


Fig 3

(10 marks)

Question 4 continues overleaf

(c)

(i) Solve the recurrence relation

$a_n = 5a_{n-1} - 6a_{n-2}$ for $n \geq 2$, $a_0 = 1$, $a_1 = 0$

(10 marks)

(ii) Find using the method of partial sums $\sum_{k=1}^{\infty} \frac{1}{2^k}$

(5 marks)

(Total 50 marks)

PLEASE ASK FOR THE NEW MATHEMATICS TABLES

Laws of Set Theory

Equivalence	***Name***
$A \cap U \Leftrightarrow A$ $A \cup \emptyset \Leftrightarrow A$	Identity Laws
$A \cap \emptyset \Leftrightarrow \emptyset$ $A \cup U \Leftrightarrow U$	Domination Laws/Bound Laws
$A \cap A \Leftrightarrow A$ $A \cup A \Leftrightarrow A$	Idempotent Laws
$(A^c)^c \Leftrightarrow A$ or $(A')' \Leftrightarrow A$	Double Negation
$A \cap B \Leftrightarrow B \cap A$ $A \cup B \Leftrightarrow B \cup A$	Commutative Laws
$(A \cap B) \cap C \Leftrightarrow A \cap (B \cap C)$ $(A \cup B) \cup C \Leftrightarrow A \cup (B \cup C)$	Associative Laws
$A \cup (B \cap C) \Leftrightarrow (A \cup B) \cap (A \cup C)$ $A \cap (B \cup C) \Leftrightarrow (A \cap B) \cup (A \cap C)$	Distributive Laws
$(A \cap B)^c \Leftrightarrow A^c \cup B^c$ or $(A \cap B)' \Leftrightarrow A' \cup B'$ $(A \cup B)^c \Leftrightarrow A^c \cap B^c$ or $(A \cup B)' \Leftrightarrow A' \cap B'$	De Morgan's Law
$A \cup A^c \Leftrightarrow U$ $A \cap A^c \Leftrightarrow \emptyset$	Complement Laws
$\emptyset^c \Leftrightarrow U$ $U^c \Leftrightarrow \emptyset$	0/1 Laws
$A \cap (A \cup B) \Leftrightarrow A$ $A \cup (A \cap B) \Leftrightarrow A$	Absorption laws

Table of Logical Equivalences Boolean Algebra symbols

Equivalence	***Name***
$p \,.\, 1 \Leftrightarrow p$ $p + 0 \Leftrightarrow p$	Identity Laws
$p + 1 \Leftrightarrow 1$ $p \times 0 \Leftrightarrow 0$	Domination Laws
$p + p \Leftrightarrow p$ $p \times p \Leftrightarrow p$	Idempotent Laws
$\overline{\overline{(p)}} \Leftrightarrow p$	Double Negation
$p + q \Leftrightarrow q + p$ $p \times q \Leftrightarrow q \times p$	Commutative Laws
$(p + q) + r \Leftrightarrow p + (q + r)$ $(p \times q) \times r \Leftrightarrow p \times (q \times r)$	Associative Laws
$p + (q \times r) \Leftrightarrow (p + q) \times (p + r)$ $p \times (q + r) \Leftrightarrow (p \times q) + (p \times r)$	Distributive Laws
$\overline{(p \times q)} \Leftrightarrow \overline{p} + \overline{q}$ $\overline{(p + q)} \Leftrightarrow \overline{p} \times \overline{q}$	De Morgan’s Law
$p + \overline{p} \Leftrightarrow 1$ $p \times \overline{p} \Leftrightarrow 0$	Complement Laws
$p \rightarrow q \Leftrightarrow \overline{p} + q$	Material Implication (MI)
$p \times (p + q) \Leftrightarrow p$ $p + p \times q \Leftrightarrow p$	Absorption Laws