Waterford Institute of Technology
Waterford

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BSc(Hons) in Applied Computing
BSc(Hons) in Computer Science
BSc(Hons) in Entertainment systems
BSc(Hons) in the Internet of things

Discrete Mathematics

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Dr.. Flynn

Answer all *four* questions(25 marks each).

Time allowed: Two hours.

Marks may be lost if not all your work is clearly shown
or if you do not indicate where a calculator has been used.

1. (a) The universal set is $\{1, 2, \ldots 10\}$. Let $A = \{1, 2, 3, 4, 6, 7\}$, $B = \{2, 3, 4, 5, 8\}$ and $C = \{1, 3, 5, 6, 8, 9\}$. Find the elements of the following sets:

   (i) $A \Delta B$

   (ii) $(A \cap B)' \backslash C$

   We used a plus enclosed in a circle for symmetric difference of sets

   (5 marks)

   (b) Given the relation R such that $R = \{(m, n) \in R | m, n \in A, m^2 - n \geq 4\}$ when A is the set $\{0, 1, 2, 3, 4\}$,

   (i) express the relation R as a set of ordered pairs.

   (ii) draw an arrow diagram to represent this relation. digraph

   (iii) investigate whether the relation is symmetric, transitive and/or reflexive.

   (10 marks)

   (c) Determine if the following functions are one-to-one and/or onto and use these to determine whether the function is a bijection. Assume both functions are such where $f : R \to R$.

   (i) $f(x) = 6x - 9$

   (ii) $f(x) = x^2 - 2x + 1$

   We spent less time on properties of functions over Real (that's for next semester)

   (10 marks)

   (Total 25 marks)

2. (a) Using a truth table, show that $(p \wedge q) \to p$ is a tautology.

   (5 marks)

   (b) Use the laws of logic(these are provided at the end of the exam paper) to show that

   $$\neg(p \vee (\neg p \wedge q)) \iff (\neg p \wedge \neg q)$$

   we spend less time on algebraic manipulation of logical expressions, instead focused on truth tables

   (10 marks)

   (c) Determine the truth value of each of the following statements if the universe is the set of all integers:

   (i) $(\forall n \in Z)(n^2 \geq 0)$.

   (ii) $(\exists n \in Z)(n^2 = 2)$.

   (iii) $(\forall n \in Z)(n^2 \geq n)$.

   (iv) $(\exists n \in Z \quad \forall m \in Z)(nm = m)$.

   (10 marks)

   (Total 25 marks)

3. (a) Find the coefficient of x^2y^7 in the expansion of $(2x + y)^9$.

(5 marks)

(b) A box contains 7 red socks and 5 green socks. Find the number of ways two socks can be drawn from the box if

(i) they can be any color.

(ii) they must be the same color.

(10 marks)

(c) Consider the second-order homogeneous recurrence relation $a_n = 2a_{n-1} + 8a_{n-2}$ with initial conditions $a_0 = 4$ and $a_1 = 10$.

(i) Find the next two terms of the sequence.

(ii) Find the general solution.

(iii) Find the unique solution with the given initial conditions.

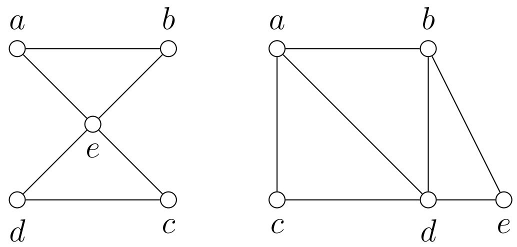
(10 marks)

(Total 25 marks)

4. (a) Draw the graph $K_{3,3}$.

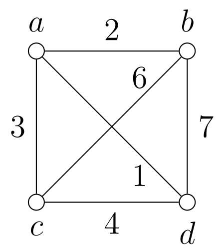
(5 marks)

(b) Consider the graphs below. Determine which graph has an Euler Path and which graph has an Euler Circuit. Give reasons for your answers.



(10 marks)

(c) Solve the Travelling Salesman Problem for the following graph by finding the length of all Hamilton circuits.



We did not cover the TSP algorithm

(10 marks)

(Total 25 marks)

Idempotent laws	(1a) $A \cup A = A$ (1b) $A \cap A = A$
Associative laws	(2a) $(A \cup B) \cup C = A \cup (B \cup C)$ (2b) $(A \cap B) \cap C = A \cap (B \cap C)$
Commutative laws	(3a) A $\cup B = B \cup A$ (3b) A $\cap B = B \cap A$
Distributive laws	(4a) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ (4b) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
Identity laws	(5a) $A \cup \emptyset = A$ (5b) $A \cap \mathbf{U} = A$ (6a) $A \cup \mathbf{U} = \mathbf{U}$ (6b) $A \cap \emptyset = \emptyset$
Involution laws	(7) $(A')' = A$
Complement laws	(8a) $A \cup A' = \mathbf{U}$ (8b) $A \cap A' = \emptyset$ (9a) U' $= \emptyset$ (9b) $\emptyset' = \mathbf{U}$
DeMorgan's laws	(10a) $(A \cup B)' = A' \cap B'$ (10b) $(A \cap B)' = A' \cup B'$

Table 1: The laws of algebra sets

Idempotent laws	(1a) $p \vee p \iff p$ (1b) $p \wedge p \iff p$
Associative laws	(2a) $(p \vee q) \vee r \iff p \vee (q \vee r)$ (2b) $(p \wedge q) \wedge r \iff p \wedge (q \wedge r)$
Commutative laws	(3a) p $\vee q \iff q \vee p$ (3b) p $\wedge q \iff q \wedge p$
Distributive laws	(4a) $p \vee (q \wedge r) \iff (p \vee q) \wedge (p \vee r)$ (4b) $p \wedge (q \vee r) \iff (p \wedge q) \vee (p \wedge r)$
Identity laws	(5a) $p \vee F \iff p$ (5b) $p \wedge T \iff p$ (6a) $p \vee T \iff T$ (6b) $p \wedge F \iff F$
Involution laws	(7) $\neg\neg p \iff p$
Complement laws	(8a) $p \vee \neg p \iff T$ (8b) $p \wedge \neg p \iff F$ (9a) $\neg T \iff F$ (9b) $\neg F \iff T$
DeMorgan's laws	(10a) $\neg(p \vee q) \iff \neg p \wedge \neg q$ (10b) $\neg(p \wedge q) \iff \neg p \vee \neg q$

Table 2: The laws of logic