Waterford Institute of Technology
Waterford

December 2015

BSc(Hons) in Applied Computing
BSc(Hons) in Computer Science
BSc(Hons) in Entertainment systems
BSc(Hons) in the Internet of things

Discrete Mathematics

Dr. A. Hennessy
Dr.. Flynn

Answer all *four* questions(25 marks each).

Time allowed: Two hours.

Marks may be lost if not all your work is clearly shown
or if you do not indicate where a calculator has been used.

1. (a) (i) $\{1, 6, 7\}$
   (ii) $\{7, 10\}$

(5 marks)

(b) (i) $R = \{(2,0), (3,0), (3,1), (3,2), (4,0), (4,1), (4,2), (4,3)\}$
(ii) An arrow diagraw from $A \to A$ showing all the ordered pairs in the set R.
(iii)
- Not symmetric. Eg $(1,3) \notin R$.
- Not reflexive. Eg $(1,1) \notin R$.
- Transitive as $\forall (a,b), (b,c) \in R, (a,c) \in R$.

(10 marks)

(c) (i) Bijection; linear function over the reals with non- zero coefficient of x.
(ii) Neither one-to-one or onto.
Eg. Not one to one as $(2,1), (0,1) \in f$, Not onto as $x \notin R$ for $y = -1$

(10 marks)

(Total 25 marks)

2. (a)

p	q	$p \wedge q$	$(p \wedge q) \to p$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	T

(5 marks)

(b) One possible solution:

$$\begin{aligned}
\neg(p \vee (\neg p \wedge q)) &\iff \neg((p \vee \neg p) \wedge (p \vee q)) \text{ (Distributive law)} \\
&\iff \neg(T \wedge (p \vee q)) \text{ (Identity law)} \\
&\iff \neg((T \wedge p) \vee (T \wedge q)) \text{ (Distributive law)} \\
&\iff \neg(p \vee q) \text{ (Identity law)} \\
&\iff \neg p \wedge \neg q \text{ (De Morgan's law)}
\end{aligned}$$

(10 marks)

(c) (i) True, integers squared always positive
(ii) False, no such integer a exists where $a^2 = 2$
(iii) True, integers squared always greater than itself.
(iv) True for $n = 1$.

(10 marks)

(Total 25 marks)

3. (a) 144

(5 marks)

(b) (i) $\binom{12}{2}$

(ii) $\binom{7}{2} + \binom{5}{2}$

(10 marks)

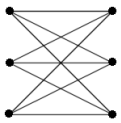
(c) (i) $a_2 = 52, a_3 = 184.$

(ii) $a_n = c_1 4^n + c_2(-2)^n.$

(iii) $a_n = 3.4^n + 1(-2)^n.$

(10 marks)

(Total 25 marks)

4. (a) 

(5 marks)

(b) Graph one has an Euler path, since there are no odd vertices. Graph two has an Euler circuit since all vertices are even.

(10 marks)

(c)
- $|ABCDA| = 2 + 6 + 4 + 2 = 13$
- $|ACBDA| = 3 + 6 + 7 + 1 = 17$
- $|ABCDA| = 3 + 4 + 7 + 2 = 16$

(10 marks)

(Total 25 marks)