WIT

COURSE LONG NAME

EXAMINATION:

MODULE NAME

SEMESTER — 1

DECEMBER 2016 DURATION: 2 HOURS

INTERNAL EXAMINAR: DR AOIFE HENNESY
DR DENIS FLYNN

EXTERNAL EXAMINAR:

DATE:
TIME:
VENUE:

INSTRUCTIONS TO CANDIDATES

1. ALL QUESTIONS CARRY EQUAL MARKS.
2. ATTEMPT ALL FOUR QUESTIONS (25 MARKS EACH).

WATERFORD INSTITUTE OF TECHNOLOGY

Question 1

(a) The universal set is $\{1, 2, 3, 4, 5, 6, \ldots, 9\}$. Let $A = \{1, 2, 5, 6\}$, $B = \{2, 5, 7\}$ and $C = \{1, 3, 5, 7, 9\}$. Find the elements of the following sets:

(i) $(A \cup C) \setminus B$

(ii) $(B \triangle C) \setminus A$

(5 marks)

(b) Given the relation R such that $R = \{(m, n) \in R | m, n \in A, (m^2 - n^2) \text{ divisible by } 4\}$ when A is the set $\{1, 3, 4, 8\}$

(i) Express the relation R as a set of ordered pairs.

(ii) Draw an arrow diagram to represent this relation.

(iii) Investigate whether the relation is symmetric, transitive and/or reflexive.

(iv) State whether the relation is an equivalence relation or not..

(10 marks)

(c) Determine if the following functions are one-to-one and/or onto and use these to determine whether the function is a bijection. Assume both functions are such where $f : R \to R$

(i) $f(x) = 3x + 1$

(ii) $f(x) = x^2 - 5$

(10 marks)

Question 2

(a) Construct the truth table for: $\neg(p \wedge q) \vee (r \wedge \neg q)$ and state whether it is a tautology, contradiction or a contingency, and why. **(5 marks)**

(b) Using the laws of logic(at the end of the exam paper) to show that:

$$\neg(p \vee q) \vee (\neg p \wedge q) \iff \neg p$$

(10 marks)

(c) Let $A = \{1, 2, 3, 4\}$. Determine the truth value of each of following statements, and explain why this is the case.

(i) $(\forall x \in A)(x + 5 < 12)$

(ii) $(\forall x \in A)(\forall y \in A)(x + y < 12)$

(iii) $(\exists x \in A)(x^2 + 2 \geq 18)$

(iv) $(\exists x \in A)(\exists y \in A)(x^2 + y^3 \leq 12)$

(10 marks)

Question 3

(a) Expand fully the binomial expansion $(x - 3y)^6$
Find the coefficient of the fourth term. **(5 marks)**

(b) **(i)** How many numbers between 500 and 1000 can be made from the digits 2,4,6,8, if no digits are repeated.

(ii) How many numbers between 500 and 1000 can be made from the digits 2,4,6,8, if digits may be repeated

(iii) Write the following series using $\sum_{r}^{\infty}$

$$1 - x + x^2 - x^3 + x^4 - \cdots .$$

(iv) Find an expression for S_n of the series: $4 + 12 + 36 + \cdots$.

(10 marks)

(c) Consider the second-order homogeneous recurrence relation $a_n = 9a_{n-1} - 18a_{n-2}$ with initial conditions $a_0 = 0$ and $a_1 = 1$.

(i) Find the next three terms of the sequence.

(ii) Find the general solution.

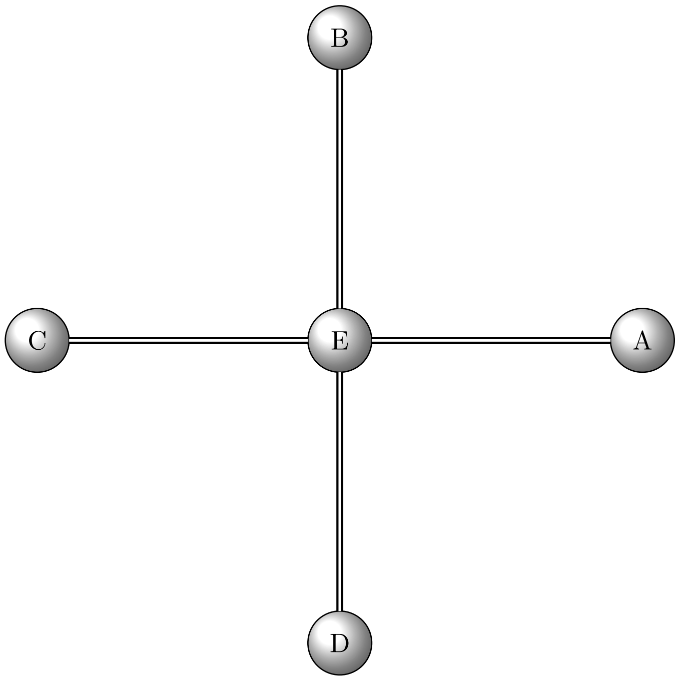
(iii) Find the unique solution with the given initial conditions.

(10 marks)

Question 4

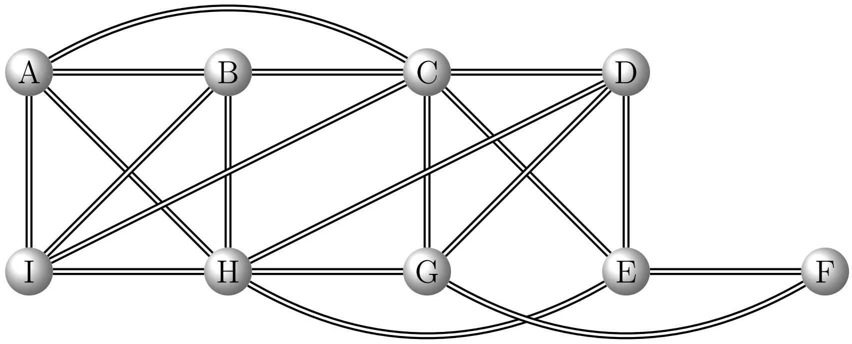
(a) **(i)** Draw the graph $K_{3,3}$

(ii) Investigate if the following graph is bipartite.

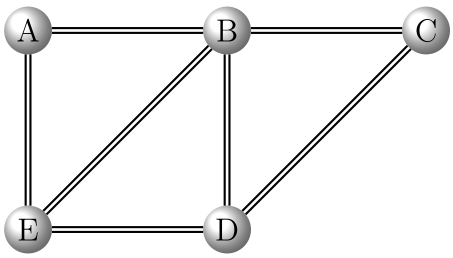


(5 marks)

(b) **(i)** Determine if there is an Euler Path or Euler Circuit in the following graph: If either an Euler path or Euler circuit exists, construct the path or circuit.

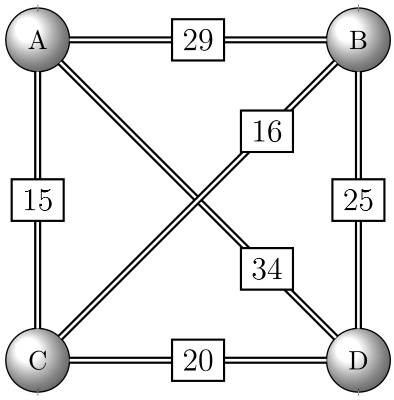


(ii) Determine whether the given graph has an Hamilton circuit. If it does, find such a circuit. It it does not, give an argument to show why no such circuit exists.



(10 marks)

(c) Solve the Travelling Salesman Problem for the following graph by using the Nearest-Neighbour Algorithm.



(10 marks)

Laws of Algebra sets

Idempotent Laws	$(1a)\ A \cup A = A$
	$(1b)\ A \cap A = A$
Associative Laws	$(2a)\ (A \cup B) \cup C = A \cup (B \cup C)$
	$(2b)\ (A \cap B) \cap C = A \cap (B \cap C)$
Commutative Laws	$(3a)\ A \cup B = B \cup A$
	$(3b)\ A \cap B = B \cap A$
Distributive Laws	$(4a)\ A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
	$(4b)\ A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
Identity Laws	$(5a)\ A \cup \emptyset = A$
	$(5b)\ A \cap \mathbb{U} = A$
	$(6a)\ A \cup \mathbb{U} = \mathbb{U}$
	$(6b)\ A \cap \emptyset = \emptyset$
Involution Laws	$(7)\ (A')' = A.$
Complement Laws	$(8a)\ A \cup A' = \mathbb{U}$
	$(8b)\ A \cap A' = \emptyset$
	$(9a)\ \mathbb{U}' = \emptyset$
	$(9b)\ \emptyset' = \mathbb{U}$
Demorgan's Laws	$(10a)\ (A \cup B)' = A' \cap B'$
	$(10b)\ (A \cap B)' = A' \cup B'$

Laws of Logic

Idempotent Laws	$(1a)\ p \vee p \iff p$
	$(1b)\ p \wedge p \iff p$
Associative Laws	$(2a)\ (p \vee q) \vee r \iff p \vee (q \vee r)$
	$(2b)\ (p \wedge q) \wedge r \iff p \wedge (q \wedge r)$
Commutative Laws	$(3a)\ p \vee q \iff q \vee p$
	$(3b)\ p \wedge q \iff q \wedge p$
Distributive Laws	$(4a)\ p \vee (q \wedge r) \iff (p \vee q) \wedge (p \vee r)$
	$(4b)\ p \wedge (q \vee r) \iff (p \wedge q) \vee (p \wedge r)$
Identity Laws	$(5a)\ p \vee F \iff p$
	$(5b)\ p \wedge T \iff p$
	$(6a)\ p \vee T \iff T$
	$(6b)\ p \wedge F \iff F$
Involution Laws	$(7)\ \neg\neg p \iff p.$
Complement Laws	$(8a)\ p \vee \neg p \iff T$
	$(8b)\ p \wedge \neg p \iff F$
	$(9a)\ \neg T \iff F$
	$(9b)\ \neg F \iff T$
Demorgan's Laws	$(10a)\ \neg(p \vee q) \iff \neg p \wedge \neg q$
	$(10b)\ \neg(p \wedge q) \iff \neg p \vee \neg q$

Question 1

(a) **(5 marks)**

2.5 marks each for (i) and (ii)

$A = \{1, 2, 5, 6\}, B = \{2, 5, 7\}, C = \{1, 3, 5, 7, 9\}$

(i) $(A \cup C) = \{1, 2, 3, 5, 6, 7, 9\}$
$(A \cup C) \setminus B = \{1, 3, 6, 9\}$

(ii) $(B \triangle C) = \{1, 2, 3, 9\}$
$(B \triangle C) \setminus A = \{3, 9\}$

(iii) $(B \triangle C) \setminus A = \{3, 9\}$

$B \cup C = \{1, 2, 3, 5, 7, 9\}$

$B \cap C = \{5, 7\}$

$(B \cup C) \setminus (B \cap C) = \{1, 2, 3, 9\}$

$B \setminus C = \{2\}$

$C \setminus B = \{1, 3, 9\}$

$(B \setminus C) \cup (C \setminus B) = \{1, 2, 3, 9\}$

$B \triangle C = \{1, 2, 3, 9\}$

(b) **(10 marks)**

2 marks for each question part

(i) R=(1, 3), (5, 5), (1, 5), (5, 1), (1, 1), (5, 3)

(ii) The relation is not symmetric because of: (1, 3), (5, 3) do not have symmetric counterparts.

(iii) The relation is not reflexive because it does not contain: (3, 3)

(iv) The relation is not antisymmetric because of: (1, 5), (5, 1)

(v) Since Reflexive: False, Symmetric: False, AntiSymmetric: False, then R is not an equivalence relation.

(c) **(10 marks)**

5 marks each question part

(i) staight line equation: Range=CoDomain $\Rightarrow$ Onto. Unique image or each element of domain $\Rightarrow$ one to one. Therefore this is a bijective function, i.e. satisfies both onto and one:one.

(ii) quadratic equation, with Range, [-5, ∞) $\neq$ CoDomain $\Rightarrow$ Not Onto. Not one to one, since there is a non unique preimage of elements in the range, e.g. preimage of 11 is either -4 or 4.

Question 2

(a) **(5 marks)**

$\frac{5}{8}$ marks each correct line

(i) Using truth table

Table 1: Truth table

p	q	r	P(p,q,r)
False	False	False	True
False	False	True	True
False	True	False	True
False	True	True	True
True	False	False	True
True	False	True	True
True	True	False	False
True	True	True	False

(b) **(10 marks)**

(i) Using tables at end of paper **2 marks each** for the following steps.

$$\begin{aligned}
\text{LHS: } & \neg(p \vee q) \vee (\neg p \wedge q) \\
\text{DeMorgan: } & (\neg p \wedge \neg q) \vee (\neg p \wedge q) \\
\text{Distrib: } & \neg p \wedge (\neg q \vee q) \\
\text{Complement: } & \neg p \wedge T \\
\text{Identity: } & \neg p
\end{aligned}$$

(c) **(10 marks)**

(i) (a) TRUE (Truth set $=A$) **2 marks**
(b) TRUE (truth set $=A \times A$) **3 marks**
(c) TRUE (Truth set $=\{4\} \neq \emptyset$) **2 marks**
(d) TRUE (Truth set $=\{(3,1),(2,1),(3,1),(1,1),(1,2)\} \neq \emptyset$). Student indicating any member of truth set - **3 marks**

Question 3

(a) **(5 marks)**

(i) **3 marks** for correct expansion. $x^6-18x^5y+135x^4y-20*27y^3x^3+(81)(85)x^2y^4-1458xy^5+729y^6$

$=x^6-18x^5y+135x^4y-540x^3y^3+685x^2y^4-243(6)xy^5+729y^6$

2 marks for correct coeff.

Fourth term coefficient: -540

x^7y^4 in the expansion of $(x+3y)^{11}$:

$\binom{11}{4}\times x^7(3y)^4=\binom{11}{4}\times 81x^7y^4=26730x^7y^4$

coeff $=26730$

(b) **(10 marks)**

2 marks each for parts (i) and(ii). **3 marks each for rest**

(i) If no repetition then, only 3 digit numbers are possible, with four possible choices $\Rightarrow 4!$

(ii) if Repetition allowed: Four choices for each number 4^3.

(iii) $\sum_{r=1}^{\infty}(-1)^{j+1}x^j$

(iv) $4(1+3+9+\cdots)=4\sum_{j=1}^{\infty}3j$

(c) **(10 marks)**

(i)

$a_2=9; a_3=63; a_4=405;$ **1 mark each**.

3 total

(ii)

$$\begin{aligned}a_n &= 9a_{n-1}-18a_{n-2} \quad \textbf{1 mark}\\ x^2 &= 9x-18 \quad \textbf{1 mark}\\ (x-6)(x-3) &= 0 \quad \textbf{0.5 mark}\\ \text{General Solution:} & \\ a_n &= c_1(6)^n+c_2(3)^n: \quad \textbf{1 mark}\end{aligned}$$

3.5 total

(iii)

Initial conditions:

$$\begin{aligned} 0 &= c_1 + c_2 \quad \boxed{\textbf{0.5 mark}} \\ 1 &= 6c_1 + 3c_2 \quad \boxed{\textbf{0.5 mark}} \\ c_1 &= \frac{1}{3} \quad \boxed{\textbf{0.5 mark}} \\ c_2 &= -\frac{1}{3} \quad \boxed{\textbf{0.5 mark}} \end{aligned}$$

Unique Solution:

$$\begin{aligned} a_n &= \frac{1}{3}(6^n - 3^n) \quad \boxed{\textbf{1 mark}} \\ a_n &= 3^{n-1}(2^n - 1) \quad \boxed{\textbf{0.5 mark}} \end{aligned}$$

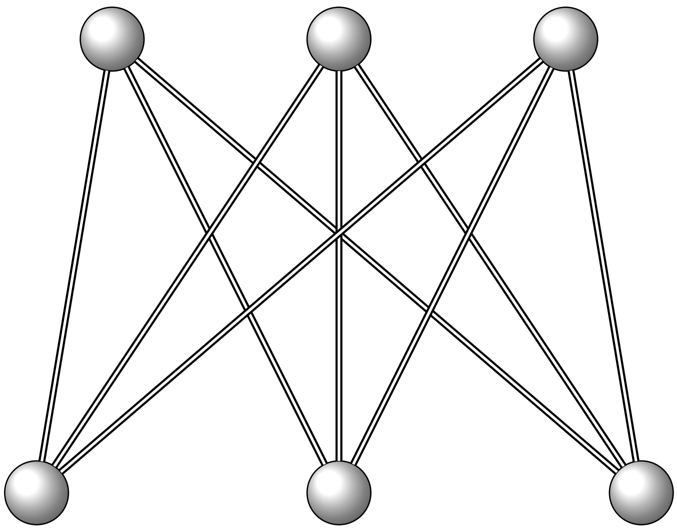
3.5 total

Question 4

(a) **(5 marks)**

3 marks first part, 2 marks for second

(i) $K_{3,3}$



(ii) Bipartite: $V_1 = \{A, B, C, D\}$ and $V_2 = \{E\}$.

(b) **(10 marks)**

6 marks and 4 marks for parts (i) and (ii) resp.

(i) From the theorem, there is an Euler circuit because every vertex has an even degree. The circuit is as follows: $\{a, b, c, d, e, f, g, h, i, a, h, b, i, c, e, h, d, g, c, a\}$

(ii) This graph has a Hamilton circuit. $\{a, b, c, d, e, a\}$ is a circuit.

(c) **(10 marks)**

(i) Student should be pick one of {A,B,C,D}
Start at A, nearest A,C,B,D cost: 15+16+25=56.
Start at B, nearest B,C,A,D cost: 16+15+34=65.
Start at C, nearest C,A,B,D cost: 15+29+25=69.
Start at D, nearest D,C,A,B cost: 20+15+29=64.
Student correctly choosing any of above paths and finding total weight = 10 marks.
Student correctly choosing any of above paths =5, correct edge weight = 1.5 x 3, correct sum 0.5.