Logic

# Discrete Mathematics

Number Theory

Mathematical Proofs

Topic 01 — Logic

Lecture 04 — Rules of Inference

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Recurrence Relations

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Set Theory

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### Outline
- Constructing arguments in propositional logic
- Normal forms

Enumeration

# Outline

## Notation

> Single-line vs Double-line Arrows

For the purpose of this module the single line arrows (representing the IFTHEN and IFF connectives)

$$\rightarrow \quad \text{and} \quad \leftrightarrow$$

mean the same thing as the corresponding double-line arrow

$$\Rightarrow \quad \text{and} \quad \Leftrightarrow$$

I will use the double-lined arrows in places where I want to treat a complicated proposition as two smaller propositions. For example, I want to think of the proposition

$$(p \rightarrow q) \wedge \neg q \implies \neg p$$

in terms of the two proposition $(p \rightarrow q) \wedge \neg q$ and $\neg p$.

## Motivation

Remember the Socrates example when we started Logic.

> *"All men are mortal.*
> *Socrates is a man.*
> *Therefore, Socrates is mortal."*

Here we have two premises:

- All men are mortal
- Socrates is a man.

and the conclusion:

- Socrates is mortal.

**Q:** How do we get the conclusion from the premises?

**A:** We construct an argument, a sequence of propositions that follow from the rules of inference until we reach the conclusion.

## Motivation

Remember the Socrates example when we started Logic.

> *"All men are mortal.*
> *Socrates is a man.*
> *Therefore, Socrates is mortal."*



Compare with this argument...
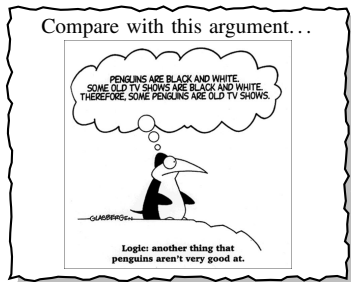
Here we have two premises:

- All men are mortal
- Socrates is a man.

and the conclusion:

- Socrates is mortal.

**Q:** How do we get the conclusion from the premises?

**A:** We construct an argument, a sequence of propositions that follow from the rules of inference until we reach the conclusion.

# Arguments

## Definition 1 (Argument)

A argument in propositional logic is a sequence of propositions. All but the final proposition are called premises. The last statement is the conclusion. The argument is valid if the premises imply the conclusion.

- If the premises are $p_1, p_2, \ldots p_n$ and the conclusion is $q$ then the argument is valid iff

$$(p_1 \wedge p_2 \wedge \cdots \wedge p_n) \rightarrow q$$

  is a tautology.
- We could use truth tables to test if an argument is valid — construct the above expression, then build the truth table and check the output column.
- Alternatively, we could sequently apply inference rules to arrive at the conclusion.
- Inference rules are simple arguments that will be used to construct more complex argument forms.

# Outline

# Detachment (Modus Ponens)

Argument

$$p \rightarrow q$$
$$\underline{\phantom{p \rightarrow} p \phantom{q}}$$
$$\therefore q$$

Corresponding Tautology

$$(p \rightarrow q) \wedge p \implies q$$

Example

Let

$p =$ "*It is snowing.*"

$q =$ "*I will study discrete maths.*"

Then the argument is

"*If it is snowing, then I will study discrete maths.*"
"*It is snowing.*"

Therefore "*I will study discrete maths.*"

# Indirect Reasoning (Modus Tollens)

Argument

$$p \to q$$
$$\neg q$$
$$\therefore \neg p$$

Corresponding Tautology

$$(p \to q) \land \neg q \implies \neg p$$

Example

Let

$p =$ "*It is snowing.*"

$q =$ "*I will study discrete maths.*"

Then the argument is

"*If it is snowing, then I will study discrete maths.*"
"*I will not study discrete maths.*"

Therefore "*It is not snowing.*"

# Chain Rule (Hypothetical Syllogism)

Argument

$$p \rightarrow q$$
$$\underline{q \rightarrow r}$$
$$\therefore p \rightarrow r$$

Corresponding Tautology

$$(p \rightarrow q) \wedge (q \rightarrow r) \implies (p \rightarrow r)$$

Example

Let

   $p = $ "*It is snowing.*"

   $q = $ "*I will study discrete maths.*"

   $r = $ "*I will get an A.*"

Then the argument is

   "*If it is snowing, then I will study discrete maths.*"
   "*If I will study discrete maths, then I will get an A.*"

   Therefore "*If it is snowing, then I will get an A.*"

# Chain Rule (Hypothetical Syllogism)

Argument

$$p \rightarrow q$$
$$q \rightarrow r$$
$$\therefore p \rightarrow r$$

Corresponding Tautology

$$(p \rightarrow q) \wedge (q \rightarrow r) \implies (p \rightarrow r)$$

Example

Let

$p =$ "*It is snowing.*"

$q =$ "*I will study discrete maths.*"

$r =$ "*I will get an A.*"

Then the argument is

"*If it is snowing, then I will study discrete maths.*"
"*If I will study discrete maths, then I will get an A.*"

Therefore "*If it is snowing, then I will get an A.*"

# Disjunctive Simplification (Disjunctive Syllogism)

Argument

$$p \vee q \qquad p \vee q$$
$$\frac{\neg p}{\therefore q} \qquad \frac{\neg q}{\therefore p}$$

Corresponding Tautology

$$(p \vee q) \wedge (\neg p) \implies q$$
$$(p \vee q) \wedge (\neg q) \implies p$$

Example

Let

    $p$ = "*I will study discrete maths.*"

    $q$ = "*I will study programming.*"

Then the argument is

    "*I will study discrete maths or I will study programming.*"

    "*I will not study discrete maths.*"

    Therefore "*I will study programming.*"

# Disjunctive Addition

Argument

$$\frac{p}{\therefore p \vee q}$$

Corresponding Tautology

$$p \implies (p \vee q)$$

Example

Let

$p = $ "*I will study discrete maths.*"

$q = $ "*I will get high.*"

Then the argument is

"*I will study discrete maths.*"

Therefore "*I will study discrete maths or I will get high.*"

# Conjunctive Simplification

Argument

$$\frac{p \wedge q}{\therefore p} \qquad \frac{p \wedge q}{\therefore q}$$

Corresponding Tautology

$$(p \wedge q) \implies p$$
$$(p \wedge q) \implies q$$

Example

Let

$p =$ *"I will study discrete maths."*

$q =$ *"I will get high."*

Then the argument is

*"I will study discrete maths and I will get high."*

Therefore *"I will study discrete maths."*

## Resolution

Argument

$$\neg p \vee r$$
$$\underline{p \vee q}$$
$$\therefore q \vee r$$

Corresponding Tautology

$$(\neg p \vee r) \wedge (p \vee q) \implies (q \vee r)$$

Example

Let

*p* = "*I will study discrete maths.*"

*p* = "*I will study programming.*"

*p* = "*I will study databases.*"

Then the argument is

"*I will not study discrete maths or I will study programming.*"
"*I will study discrete maths or I will study databases.*"

Therefore "*I will study programming or I will study databases.*"

# Outline

# Example 2 I

A valid argument is a sequence of statements. Each statement is either a premise or follows from previous statements by rules of inference. The last statement is called conclusion.

### Example 2

Assuming the following two propositions

$$p \quad \text{and} \quad (p \to q)$$

show that $q$ is a conclusion.

> Method 1

Construct argument using inference rules …

|  | Step | Reason |
|---|---|---|
| 1) | $p \land (p \to q)$ | Premise |
| 2) | $p$ | Conjunctive Simplification from (1) |
| 3) | $p \to q$ | Conjunctive Simplification from (1) |
| $\therefore$ | $q$ | Detachment (Modus Ponens) from (2) and (3) |

## Example 2 II

> Method 2

Construct an expression of the form

(premise 1) $\land$ (premise 2) $\land \cdots \land$ (premise $n$) $\implies$ (conclusion)

and verify that the expression is a tautology (using a truth table).

So for this example ...

$$\underbrace{(\text{premise 1})}_{p} \land \underbrace{(\text{premise 2})}_{(p \to q)} \implies \underbrace{(\text{conclusion})}_{q}$$

| inputs | | individual premises | | argument premise | argument conclusion | argument |
|--------|---|---|---|---|---|---|
| $p$ | $q$ | $p$ | $(p \to q)$ | $p \land (p \to q)$ | $q$ | $p \land (p \to q) \Rightarrow q$ |
| F | F | F | T | F | F | T |
| F | T | F | T | F | T | T |
| T | F | T | F | F | F | T |
| T | T | T | T | T | T | T |

# Example 2                                                                    II

> Method 2

Construct an expression of the form

(premise 1) $\land$ (premise 2) $\land \cdots \land$ (premise $n$) $\implies$ (conclusion)

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|---|---|---|---|---|---|---|
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| F | F | F | T | F | F | T |
| F | T | F | T | F | T | T |
| T | F | T | F | F | F | T |
| T | T | T | T | T | T | T |

## Example 2                                                                    II

⟩Method 2⟩

Construct an expression of the form

$$(\text{premise } 1) \wedge (\text{premise } 2) \wedge \cdots \wedge (\text{premise } n) \implies (\text{conclusion})$$

and verify that the expression is a tautology (using a truth table).
So for this example . . .

$$\underbrace{(\text{premise } 1)}_{p} \wedge \underbrace{(\text{premise } 2)}_{(p \to q)} \implies \underbrace{(\text{conclusion})}_{q}$$

| inputs | | individual premises | | argument premise | argument conclusion | argument |
|---|---|---|---|---|---|---|
| $p$ | $q$ | $p$ | $(p \to q)$ | $p \wedge (p \to q)$ | $q$ | $p \wedge (p \to q) \Rightarrow q$ |
| F | F | F | T | F | F | T |
| F | T | F | T | F | T | T |
| T | F | T | F | F | F | T |
| T | T | T | T | T | T | T |

## Example 2                                                                                              II

$\rangle$ Method 2 $\rangle$

Construct an expression of the form

(premise 1) $\land$ (premise 2) $\land \cdots \land$ (premise $n$) $\implies$ (conclusion)

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| inputs | | individual premises | | argument premise | argument conclusion | argument |
|---|---|---|---|---|---|---|
| $p$ | $q$ | $p$ | $(p \to q)$ | $p \land (p \to q)$ | $q$ | $p \land (p \to q) \Rightarrow q$ |
| **F** | **F** | F | T | F | F | T |
| **F** | **T** | F | T | F | T | T |
| **T** | **F** | T | F | F | F | T |
| **T** | **T** | T | T | T | T | T |

# Example 2 II

> Method 2

Construct an expression of the form

(premise 1) $\wedge$ (premise 2) $\wedge \cdots \wedge$ (premise $n$) $\implies$ (conclusion)

and verify that the expression is a tautology (using a truth table).
So for this example ...

$$\underbrace{(\text{premise 1})}_{p} \wedge \underbrace{(\text{premise 2})}_{(p \to q)} \implies \underbrace{(\text{conclusion})}_{q}$$

| inputs | | individual premises | | argument premise | argument conclusion | argument |
|---|---|---|---|---|---|---|
| $p$ | $q$ | $p$ | $(p \to q)$ | $p \wedge (p \to q)$ | $q$ | $p \wedge (p \to q) \Rightarrow q$ |
| **F** | **F** | **F** | **T** | F | F | T |
| **F** | **T** | **F** | **T** | F | T | T |
| **T** | **F** | **T** | **F** | F | F | T |
| **T** | **T** | **T** | **T** | T | T | T |

# Example 2                                                                              II

> Method 2

Construct an expression of the form

$$\text{(premise 1)} \land \text{(premise 2)} \land \cdots \land \text{(premise } n\text{)} \implies \text{(conclusion)}$$

and verify that the expression is a tautology (using a truth table).
So for this example ...

$$\underbrace{\text{(premise 1)}}_{p} \land \underbrace{\text{(premise 2)}}_{(p \to q)} \implies \underbrace{\text{(conclusion)}}_{q}$$

| inputs | | individual premises | | argument premise | argument conclusion | argument |
|--------|--------|--------|--------|--------|--------|--------|
| $p$ | $q$ | $p$ | $(p \to q)$ | $p \land (p \to q)$ | $q$ | $p \land (p \to q) \Rightarrow q$ |
| **F** | **F** | **F** | **T** | **F** | **F** | T |
| **F** | **T** | **F** | **T** | **F** | **T** | T |
| **T** | **F** | **T** | **F** | **F** | **F** | T |
| **T** | **T** | **T** | **T** | **T** | **T** | T |

# Example 2                                                                 II

> Method 2

Construct an expression of the form

$$(\text{premise } 1) \wedge (\text{premise } 2) \wedge \cdots \wedge (\text{premise } n) \implies (\text{conclusion})$$

and verify that the expression is a tautology (using a truth table).
So for this example ...

$$\underbrace{(\text{premise } 1)}_{p} \wedge \underbrace{(\text{premise } 2)}_{(p \to q)} \implies \underbrace{(\text{conclusion})}_{q}$$

| inputs | | individual premises | | argument premise | argument conclusion | argument |
|---|---|---|---|---|---|---|
| $p$ | $q$ | $p$ | $(p \to q)$ | $p \wedge (p \to q)$ | $q$ | $p \wedge (p \to q) \Rightarrow q$ |
| **F** | **F** | **F** | **T** | **F** | **F** | **T** |
| **F** | **T** | **F** | **T** | **F** | **T** | **T** |
| **T** | **F** | **T** | **F** | **F** | **F** | **T** |
| **T** | **T** | **T** | **T** | **T** | **T** | **T** |

# Example 3 I

### Example 3

With these hypotheses:

- ⓐ *"It is not sunny this afternoon and it is colder than yesterday."*
- ⓑ *"We will go swimming only if it is sunny."*
- ⓒ *"If we do not go swimming, then we will take a canoe trip."*
- ⓓ *"If we take a canoe trip, then we will be home by sunset."*

Using the inference rules, construct a valid argument for the conclusion:

- *"We will be home by sunset."*

General procedure …

(STEP 1) Choose propositional variables.

(STEP 2) Translation into propositional logic.

(STEP 3) Construct the valid argument (OR verify related tautology using truth table.)

## Example 3                                              I

### Example 3

With these hypotheses:

- ⓐ *"It is not sunny this afternoon and it is colder than yesterday."*
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- ⓓ *"If we take a canoe trip, then we will be home by sunset."*

Using the inference rules, construct a valid argument for the conclusion:

- *"We will be home by sunset."*

General procedure …

(STEP 1) Choose propositional variables.

(STEP 2) Translation into propositional logic.

(STEP 3) Construct the valid argument (OR verify related tautology using truth table.)

# Example 3                                                                II

(STEP 1) *Choose propositional variables.*

- $s =$ *"It is Sunny this afternoon."*
- $c =$ *"It is Colder than yesterday."*
- $w =$ *"We will go sWimming"*
- $t =$ *"We will take a canoe Trip."*
- $h =$ *"We will be Home by sunset."*

(STEP 2) *Translation into propositional logic.*

Premises . . .

a) *"It is not sunny this afternoon and it is colder than yesterday."*    $\neg s \wedge c$
b) *"We will go swimming only if it is sunny."*                          $w \rightarrow s$
c) *"If we do not go swimming, then we will take a canoe trip."*          $\neg w \rightarrow t$
d) *"If we take a canoe trip, then we will be home by sunset."*          $t \rightarrow h$

and conclusion

- *"We will be home by sunset."*                                          $h$

# Example 3 II

$\boxed{\text{Step 1}}$ *Choose propositional variables.*

- $s =$ "*It is Sunny this afternoon.*"
- $c =$ "*It is Colder than yesterday.*"
- $w =$ "*We will go sWimming*"
- $t =$ "*We will take a canoe Trip.*"
- $h =$ "*We will be Home by sunset.*"

$\boxed{\text{Step 2}}$ *Translation into propositional logic.*

Premises . . .

- **a)** "*It is not sunny this afternoon and it is colder than yesterday.*"  $\quad \neg s \wedge c$
- **b)** "*We will go swimming only if it is sunny.*"  $\quad w \rightarrow s$
- **c)** "*If we do not go swimming, then we will take a canoe trip.*"  $\quad \neg w \rightarrow t$
- **d)** "*If we take a canoe trip, then we will be home by sunset.*"  $\quad t \rightarrow h$

and conclusion

- "*We will be home by sunset.*"  $\quad h$

# Example 3 III

$\boxed{\text{STEP 3}}$ *Construct the valid argument*     *(Note the truth table here would have 32 rows)*

We have

| | Premises | | | Conclusion |
|:---:|:---:|:---:|:---:|:---:|
| (a) | (b) | (c) | (d) | |
| $\neg s \wedge c$ | $w \rightarrow s$ | $\neg w \rightarrow t$ | $t \rightarrow h$ | $h$ |

And our argument is ...

| | Step | Reason |
|:---:|:---|:---|
| 1) | $\neg s \wedge c$ | Premise (a) |
| 2) | $\neg s$ | Conjunctive Simplification from (1) |
| 3) | $w \rightarrow s$ | Premise (b) |
| 4) | $\neg w$ | Indirect Reasoning (Modus Tollens) from (2) and (3) |
| 5) | $\neg w \rightarrow t$ | Premise (c) |
| 6) | $t$ | Detachment (Modus Ponens) from (4) and (5) |
| 7) | $t \rightarrow h$ | Premise (d) |
| $\therefore$ | $h$ | Detachment (Modus Ponens) from (6) and (7) |

# Example 3 III

STEP 3 *Construct the valid argument* *(Note the truth table here would have 32 rows)*

We have

|  | Premises |  |  | Conclusion |
|---|---|---|---|---|
| (a) | (b) | (c) | (d) |  |
| $\neg s \wedge c$ | $w \rightarrow s$ | $\neg w \rightarrow t$ | $t \rightarrow h$ | $h$ |

And our argument is ...

|  | **Step** | **Reason** |
|---|---|---|
| 1) | $\neg s \wedge c$ | Premise (a) |
| 2) | $\neg s$ | Conjunctive Simplification from (1) |
| 3) | $w \rightarrow s$ | Premise (b) |
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| 7) | $t \rightarrow h$ | Premise (d) |
| $\therefore$ | $h$ | Detachment (Modus Ponens) from (6) and (7) |

# Example 3 III

STEP 3  *Construct the valid argument*　　　*(Note the truth table here would have 32 rows)*

We have

|  | Premises |  |  | Conclusion |
|---|---|---|---|---|
| (a) | (b) | (c) | (d) |  |
| $\neg s \wedge c$ | $w \rightarrow s$ | $\neg w \rightarrow t$ | $t \rightarrow h$ | $h$ |

And our argument is …

|  | **Step** | **Reason** |
|---|---|---|
| 1) | $\neg s \wedge c$ | Premise (a) |
| 2) | $\neg s$ | Conjunctive Simplification from (1) |
| 3) | $w \rightarrow s$ | Premise (b) |
| 4) | $\neg w$ | Indirect Reasoning (Modus Tollens) from (2) and (3) |
| 5) | $\neg w \rightarrow t$ | Premise (c) |
| 6) | $t$ | Detachment (Modus Ponens) from (4) and (5) |
| 7) | $t \rightarrow h$ | Premise (d) |
| ∴ | $h$ | Detachment (Modus Ponens) from (6) and (7) |

# Example 3 III

STEP 3 *Construct the valid argument* *(Note the truth table here would have 32 rows)*

We have

|  | Premises |  |  | Conclusion |
|---|---|---|---|---|
| (a) | (b) | (c) | (d) | |
| $\neg s \wedge c$ | $w \rightarrow s$ | $\neg w \rightarrow t$ | $t \rightarrow h$ | $h$ |

And our argument is . . .

|  | **Step** | **Reason** |
|---|---|---|
| 1) | $\neg s \wedge c$ | Premise (a) |
| 2) | $\neg s$ | Conjunctive Simplification from (1) |
| 3) | $w \rightarrow s$ | Premise (b) |
| 4) | $\neg w$ | Indirect Reasoning (Modus Tollens) from (2) and (3) |
| 5) | $\neg w \rightarrow t$ | Premise (c) |
| 6) | $t$ | Detachment (Modus Ponens) from (4) and (5) |
| 7) | $t \rightarrow h$ | Premise (d) |
| $\therefore$ | $h$ | Detachment (Modus Ponens) from (6) and (7) |

# Example 3                                                                III

$\boxed{\text{STEP 3}}$ *Construct the valid argument*          *(Note the truth table here would have 32 rows)*

We have

|  | Premises |  |  | Conclusion |
|---|---|---|---|---|
| (a) | (b) | (c) | (d) |  |
| $\neg s \wedge c$ | $w \to s$ | $\neg w \to t$ | $t \to h$ | $h$ |

And our argument is . . .

|  | **Step** | **Reason** |
|---|---|---|
| 1) | $\neg s \wedge c$ | Premise (a) |
| 2) | $\neg s$ | Conjunctive Simplification from (1) |
| 3) | $w \to s$ | Premise (b) |
| 4) | $\neg w$ | Indirect Reasoning (Modus Tollens) from (2) and (3) |
| 5) | $\neg w \to t$ | Premise (c) |
| 6) | $t$ | Detachment (Modus Ponens) from (4) and (5) |
| 7) | $t \to h$ | Premise (d) |
| $\therefore$ | $h$ | Detachment (Modus Ponens) from (6) and (7) |

# Example 3                                                                                III

$\boxed{\text{STEP 3}}$ *Construct the valid argument*        *(Note the truth table here would have 32 rows)*

We have

|  | Premises |  |  | Conclusion |
|:---:|:---:|:---:|:---:|:---:|
| (a) | (b) | (c) | (d) | |
| $\neg s \wedge c$ | $w \rightarrow s$ | $\neg w \rightarrow t$ | $t \rightarrow h$ | $h$ |

And our argument is …

|  | **Step** | **Reason** |
|:---:|:---|:---|
| 1) | $\neg s \wedge c$ | Premise (a) |
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| 7) | $t \rightarrow h$ | Premise (d) |
| $\therefore$ | $h$ | Detachment (Modus Ponens) from (6) and (7) |

# Example 3 III

STEP 3  *Construct the valid argument*   *(Note the truth table here would have 32 rows)*
We have

| | | Premises | | Conclusion |
|---|---|---|---|---|
| (a) | (b) | (c) | (d) | |
| $\neg s \wedge c$ | $w \to s$ | $\neg w \to t$ | $t \to h$ | $h$ |

And our argument is ...

| | Step | Reason |
|---|---|---|
| 1) | $\neg s \wedge c$ | Premise (a) |
| 2) | $\neg s$ | Conjunctive Simplification from (1) |
| 3) | $w \to s$ | Premise (b) |
| 4) | $\neg w$ | Indirect Reasoning (Modus Tollens) from (2) and (3) |
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| 6) | $t$ | Detachment (Modus Ponens) from (4) and (5) |
| 7) | $t \to h$ | Premise (d) |
| $\therefore$ | $h$ | Detachment (Modus Ponens) from (6) and (7) |

# Example 3 III

STEP 3 *Construct the valid argument* *(Note the truth table here would have 32 rows)*

We have

| | Premises | | | Conclusion |
|---|---|---|---|---|
| (a) | (b) | (c) | (d) | |
| $\neg s \wedge c$ | $w \to s$ | $\neg w \to t$ | $t \to h$ | $h$ |

And our argument is ...

| | Step | Reason |
|---|---|---|
| 1) | $\neg s \wedge c$ | Premise (a) |
| 2) | $\neg s$ | Conjunctive Simplification from (1) |
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| 7) | $t \to h$ | Premise (d) |
| $\therefore$ | $h$ | Detachment (Modus Ponens) from (6) and (7) |

# Example 3 III

STEP 3 *Construct the valid argument* *(Note the truth table here would have 32 rows)*

We have

|  | Premises |  |  | Conclusion |
|---|---|---|---|---|
| (a) | (b) | (c) | (d) |  |
| $\neg s \wedge c$ | $w \rightarrow s$ | $\neg w \rightarrow t$ | $t \rightarrow h$ | $h$ |

And our argument is ...

|  | **Step** | **Reason** |
|---|---|---|
| 1) | $\neg s \wedge c$ | Premise (a) |
| 2) | $\neg s$ | Conjunctive Simplification from (1) |
| 3) | $w \rightarrow s$ | Premise (b) |
| 4) | $\neg w$ | Indirect Reasoning (Modus Tollens) from (2) and (3) |
| 5) | $\neg w \rightarrow t$ | Premise (c) |
| 6) | $t$ | Detachment (Modus Ponens) from (4) and (5) |
| 7) | $t \rightarrow h$ | Premise (d) |
| $\therefore$ | $h$ | Detachment (Modus Ponens) from (6) and (7) |

# That was a bit painful ... let Python do the work ...        I

|  | | Premises | | | Conclusion |
|---|---|---|---|---|---|
| | (a) | (b) | (c) | (d) | |
| | $\neg s \wedge c$ | $w \to s$ | $\neg w \to t$ | $t \to h$ | $h$ |

```python
 1 # individual premises
 2 p1 = "not s and c"
 3 p2 = "not w or s"
 4 p3 = "w or t"
 5 p4 = "not t or h"
 6
 7 # construct argument premise - each premise is inside ( )
 8 p = f"({p1}) and ({p2}) and ({p3}) and ({p4})"
 9
10 # argument conclusion
11 c = "h"
12
13 # output argument premise and conclusion
14 print(f"argument premise: {p}")
15 print(f"argument conclusion: {c}")
16
17 # build expression for testing (is it a tautology?)
18 argument = f"(not ({p})) or ({c})"
19
20 # generate truth table - show premises, conclusion and argument
21 TruthTable([p1,p2,p3,p4, c, argument])
```

# That was a bit painful ... let Python do the work ...   II

```
argument premise: (not s and c) and (not w or s) and (w or t) and (not t or h)
argument conclusion: h
```

| c | h | s | t | w | not s and c | not w or s | w or t | not t or h | h | (not ((not s and c) and (not w or s) and (w or t) and (not t or h))) or (h) |
|---|---|---|---|---|---|---|---|---|---|---|
| False | False | False | False | False | False | True | False | True | False | True |
| False | False | False | False | True | False | False | True | True | False | True |
| False | False | False | True | False | False | True | True | False | False | True |
| False | False | False | True | True | False | False | True | False | False | True |
| False | False | True | False | False | False | True | False | True | False | True |
| False | False | True | False | True | False | True | True | True | False | True |
| False | False | True | True | False | False | True | True | False | False | True |
| False | False | True | True | True | False | True | True | False | False | True |
| False | True | False | False | False | False | True | False | True | True | True |
| False | True | False | False | True | False | False | True | True | True | True |
| False | True | False | True | False | False | True | True | True | True | True |
| False | True | False | True | True | False | False | True | True | True | True |
| False | True | True | False | False | False | True | False | True | True | True |
| False | True | True | False | True | False | True | True | True | True | True |
| False | True | True | True | False | False | True | True | True | True | True |
| False | True | True | True | True | False | True | True | True | True | True |
| True | False | False | False | False | True | True | False | True | False | True |
| True | False | False | False | True | True | False | True | True | False | True |
| True | False | False | True | False | True | True | True | False | False | True |
| True | False | False | True | True | True | False | True | False | False | True |
| True | False | True | False | False | False | True | False | True | False | True |
| True | False | True | False | True | False | True | True | True | False | True |
| True | False | True | True | False | False | True | True | False | False | True |
| True | False | True | True | True | False | True | True | False | False | True |
| True | True | False | False | False | True | True | False | True | True | True |
| True | True | False | False | True | True | False | True | True | True | True |
| True | True | False | True | False | True | True | True | True | True | True |
| True | True | False | True | True | True | False | True | True | True | True |
| True | True | True | False | False | False | True | False | True | True | True |
| True | True | True | False | True | False | True | True | True | True | True |
| True | True | True | True | False | False | True | True | True | True | True |
| True | True | True | True | True | False | True | True | True | True | True |

## That was a bit painful ... let Python do the work ...     II

```
argument premise: (not s and c) and (not w or s) and (w or t) and (not t or h)
argument conclusion: h
```

| c | h | s | t | w | not s and c | not w or s | w or t | not t or h | h | (not ((not s and c) and (not w or s) and (w or t) and (not t or h))) or (h) |
|---|---|---|---|---|---|---|---|---|---|---|
| False | False | False | False | False | False | True | False | True | False | True |
| False | False | False | False | True | False | False | True | True | False | True |
| False | False | False | True | False | False | True | True | False | False | True |
| False | False | False | True | True | False | False | True | False | False | True |
| False | False | True | False | False | False | True | False | True | False | True |
| False | False | True | False | True | False | True | True | True | False | True |
| False | False | True | True | False | False | True | True | False | False | True |
| False | False | True | True | True | False | True | True | False | False | True |
| False | True | False | False | False | False | True | False | True | True | True |
| False | True | False | False | True | False | False | True | True | True | True |
| False | True | False | True | False | False | True | True | True | True | True |
| False | True | False | True | True | False | False | True | True | True | True |
| False | True | True | False | False | False | True | False | True | True | True |
| False | True | True | False | True | False | True | True | True | True | True |
| False | True | True | True | False | False | True | True | True | True | True |
| False | True | True | True | True | False | True | True | True | True | True |
| True | False | False | False | False | True | True | False | True | False | True |
| True | False | False | False | True | True | False | True | True | False | True |
| True | False | False | True | False | True | True | True | False | False | True |
| True | False | False | True | True | True | False | True | False | False | True |
| True | False | True | False | False | False | True | False | True | False | True |
| True | False | True | False | True | False | True | True | True | False | True |
| True | False | True | True | False | False | True | True | False | False | True |
| True | False | True | True | True | False | True | True | False | False | True |
| True | True | False | False | False | True | True | False | True | True | True |
| True | True | False | False | True | True | False | True | True | True | True |
| True | True | False | True | False | True | True | True | True | True | True |
| True | True | False | True | True | True | False | True | True | True | True |
| True | True | True | False | False | False | True | False | True | True | True |
| True | True | True | False | True | False | True | True | True | True | True |
| True | True | True | True | False | False | True | True | True | True | True |
| True | True | True | True | True | False | True | True | True | True | True |

All rows are True so we have a tautology