Discrete Mathematics — Tutorial Sheet 01 — Logic

BSc (H) in App Comp, Ent Sys, Comp Foren, and the IoT

Translating between English and symbols

Question 1

Given propositions p = "*Jack passed math*", and q= "*Jill passed math*".

(a) Translate "Jack and Jill both passed math" into symbols.

(b) Translate "If Jack passed math, then Jill did not" into symbols.

(c) Translate "$p \vee q$" into English.

(d) Translate "$\neg(p \wedge q) \rightarrow q$" into English.

(e) Suppose you know that if Jack passed math, then so did Jill. What can you conclude if you know that:

i) Jill passed math?

ii) Jill did not pass math?

Question 2

Consider the statement "*If Oscar eats Chinese food, then he drinks milk*".

(a) Write the converse of the statement.

(b) Write the contrapositive of the statement.

(c) Is it possible for the contrapositive to be false? If it was, what would that tell you?

(d) Suppose the original statement is true, and that Oscar drinks milk. Can you conclude anything (about his eating Chinese food)? Explain.

(e) Suppose the original statement is true, and that Oscar does not drink milk. Can you conclude anything (about his eating Chinese food)? Explain.

Question 3

Let d = "*I like discrete mathematics*", c = "*I will pass this module*" and s = "*I will do my assignments*". Express each of the following propositions in symbolic form:

(a) I like discrete mathematics and I will pass this module.

(b) I will do my assignments or I will not pass this module.

(c) It is not true that I like discrete mathematics and I will do my assignments.

(d) I will not do my assignment and I will not pass this module.

Truth Tables

Question 4

Construct the truth tables of each of the following and classify them as satisfiable, tautology, or a contradiction.

(a) $\neg(p \wedge q)$ **(b)** $p \wedge (\neg q)$ **(c)** $(p \wedge q) \wedge r$

(d) $(p \wedge q) \vee (q \wedge r) \vee (r \wedge p)$ **(e)** $\neg p \vee \neg q$ **(f)** $p \vee q \vee r \vee s$

Question 5

Construct a truth table to determine whether $(\neg p \vee q) \wedge (q \rightarrow (\neg r \wedge \neg p)) \wedge (p \vee r)$ is satisfiable.

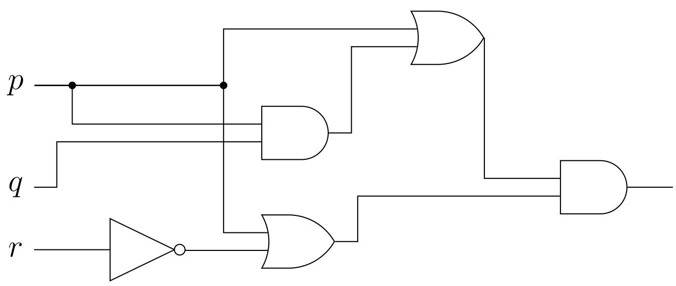
Question 6

Use the truth tables method to determine whether $p \rightarrow (q \wedge \neg q)$ and $\neg p$ are logically equivalent.

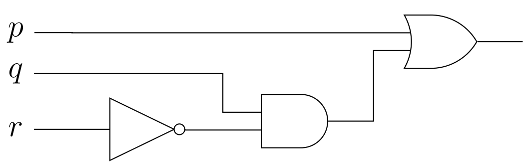
Application to logic circuits

Question 7

(a) Construct the logical expression for the given logical circuit.



(b) Construct the logical expression for the given logical circuit.



(c) Check if the two logical expressions are equivalent, and hence comment on whether the above two circuits are equivalent.

Puzzles

Question 8

Three boxes are presented to you. One contains gold, the other two are empty. Each box has imprinted on it a clue as to its contents; the clues are:

Box 1 "*The gold is not here*"

Box 2 "*The gold is not here*"

Box 3 "*The gold is in Box 2*"

Only one message is true; the other two are false. Which box has the gold?

Qualifiers

Question 9

Translate into symbols each of the following. Use $E(x)$ for "x is even" and $O(x)$ for "x is odd."

(a) No number is both even and odd.

(b) One more than any even number is an odd number.

(c) There is prime number that is even.

(d) Between any two numbers there is a third number.

(e) There is no number between a number and one more than that number.

Question 10

Translate into English each of the following

(a) $\forall x \ \big[E(x) \to E(x+2)\big]$.

(b) $\forall x \exists y \ \big[\sin(x) = y\big]$.

(c) $\forall y \exists x \ \big[\sin(x) = y\big]$.

(d) $\forall x \forall y \ \big[x^3 = y^3 \to x = y\big]$.

Question 11

Over the real numbers, use quantifiers to say that the equation $a+x=b$ has a solution for all values of a and b.

(Hint: You will need three qualifiers.)

Question 12

For each of the statements below, give a domain of discourse for which the statement is true, and a domain for which the statement is false.

(a) $\forall x \exists y \ \big[y^2 = x\big]$.

(b) $\forall x \forall y \exists z \ [x < z < y]$.

(c) $\exists x \forall y \forall z \ \big[(y < z) \Rightarrow (y \leq x \leq z)\big]$

Question 13

Suppose $P(x)$ is some predicate for which the statement $\forall x \ \big[P(x)\big]$ is true. Is it also the case that $\exists x \ \big[P(x)\big]$ is true? In other words, is the statement $\forall x \ \big[P(x)\big] \to \exists x \ \big[P(x)\big]$ always true? Is the converse always true? Explain.

Question 14

What about my three trolls?

Arguments

Question 15

Which of the following arguments are valid?

Hint: Construct the corresponding expression and check that is a tautology.

(a) "*If it is raining, it is not cold*"
"*If is not raining, John is not wearing a coat It is cold*"

∴ "*John is not wearing a coat*"

(b) "*If it is raining, it is not cold*"
"*If is not raining, John is not wearing a coat*"
"*It is cold*"

∴ "*John is not wearing a coat*"

(c) "*If it is snowing, it is cold or it is wet*"
"*If it is cold, John is wearing a coat*"
"*It is snowing*"

∴ "*John is wearing a coat*"