# Discrete Mathematics — Tutorial Sheet 03 — Sets

BSc (H) in App Comp, Ent Sys, Comp Foren, and the IoT

## Set Operations

**Question 1**

Let $A = \{0, 2, 3\}$, $B = \{2, 3\}$, $C = \{1, 5, 9\}$, $D = \{3, 2\}$, and $E = \{2, 3, 2\}$. and let the universal set be $U = \{0, 1, 2, ..., 9\}$.

**(a)** Determine:

**(i)** $A \cap B$ **(iv)** $A \cup C$ **(vii)** $\overline{A}$ **(x)** $A \oplus B$

**(ii)** $A \cup B$ **(v)** $A \setminus B$ **(viii)** $\overline{C}$

**(iii)** $B \cup A$ **(vi)** $B \setminus A$ **(ix)** $A \cap C$

**(b)** Determine which of the following are true. Give reasons for your decisions

**(i)** $A = B$ **(iv)** $E = D$ **(vii)** $A \setminus B = B \setminus A$

**(ii)** $B = C$ **(v)** $A \cap B = B \cap A$

**(iii)** $B = D$ **(vi)** $A \cup B = B \cup A$ **(viii)** $A \oplus B = B \oplus A$

**Question 2**

Let $U = \{1, 2, 3, ..., 9\}$. Give examples of sets $A$, $B$, and $C$ for which:

**(a)** $A \cap (B \cap C) = (A \cap B) \cap C$ **(d)** $A \cup A^c = U$

**(b)** $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ **(e)** $A \subseteq A \cup B$

**(c)** $(A \cup B)^c = A^c \cap B^c$ **(f)** $A \cap B \subseteq A$

Note: I used alternative notation here for complement: $A^c = \overline{A}$. This did not come to light until after the first tutorial, so rather than correcting the question, I have included this note.

**Question 3**

Draw a Venn diagram to represent each of the following:

**(a)** $A \cup \bar{B}$ **(d)** $(A \cap B) \cup C$

**(b)** $(A \bar{\cup} B)$ **(e)** $\bar{A} \cap B \cap \bar{C}$

**(c)** $A \cap (B \cup C)$ **(f)** $(A \cup B) \setminus C$

**Question 4**

Construct an example of sets $A$ and $B$ such that $A \cap B = \{3, 5\}$ and $A \cup B = \{2, 3, 5, 7, 8\}$.

**Question 5**

Construct an example of sets $A$ and $B$ such that $A \subseteq B$ and $A \in B$.

## Indirect Questions

This questions are based on set relationships and set operations also but may require a little more thought.

### Question 6

Let $U = \{1, 2, 3, ..., 9\}$. Give examples to illustrate the following facts:

**(a)** If $A \subseteq B$ and $B \subseteq C$, then $A \subseteq C$.

**(b)** There are sets $A$ and $B$ such that $A \setminus B \neq B \setminus A$

**(c)** If $U = A \cup B$ and A $\cap$ B $= \emptyset$, it always follows that $A = U \setminus B$.

**(d)** $A \oplus (B \cap C) = (A \oplus B) \cap (A \oplus C)$

### Question 7

Suppose that $U$ is an infinite universal set, and $A$ and $B$ are infinite subsets of $U$. Answer the following questions with a brief explanation.

**(a)** Must $\overline{A}$ be finite?

**(b)** Must $A \cup B$ infinite?

**(c)** Must $A \cap B$ be infinite?