Chapter Summary — Relations and Functions

This document is a very rough summary of the the concepts and tasks that we covered in this chapter. The plan is to write a similar document at the end of each chapter, but time will tell.
I hope this document will be use to you when revising the chapter. However, please do not think that this comes with any guarantee of completeness — the probability of me overlooking something is large. Please ask if you think I have omitted anything.
— kmurphy, 23 Oct, 2021

Section A: Concepts

A.1: Relations .....

- ☐ **Definition of relation based on subsets of a Cartesian product**
  - ☐ A relation is a set so properties/concepts of sets carry over to relations.
  - ☐ Terminology: source and target, and domain and image
- ☐ **Properties of relations from set A to set B (i.e., relating to the output values)**
  - ☐ one to one (injective)
  - ☐ into vs onto (surjective)
  - ☐ bijective = injective + surjective
- ☐ **Properties of relations on a set (source=target)**
  - ☐ Main three properties: reflexive, symmetric and transitive
  - ☐ anti-symmetric
  - ☐ iireflective and asymmetric
- ☐ **Equivalence relation = reflexive, symmetric and transitive**
  - ☐ Decomposition of a set into equivalence classes.
- ☐ **Representation of relations**
  - ☐ Set of ordered pairs
  - ☐ Venn diagrams — good for discrete (any usually finite) sets
  - ☐ Digraph — for relations on a set (source=target)

A.2: Functions .....

- ☐ **Definition of function as a restricted relation — exactly one outgoing arrow for each element in the source**
  - ☐ A function is a relations so properties/concepts of relations carry over to functions.
  - ☐ Formal vs informal definition of functions
- ☐ **Representation of functions (in addition to those for relations (above))**
  - ☐ Lookup table
  - ☐ Formula — good for continuous or infinite sets
  - ☐ 2D Cartesian Plots — good for continuous or infinite sets
- ☐ **Algebra of functions**
  - ☐ Notation: addition/subtraction/multiplication/division of functions
  - ☐ Function composition
    - ☐ Repeated iteration of functions
- ☐ **Function inverse**
  - ☐ bijective = necessary and sufficient condition for existence of inverse function pair.

Section B: Tasks

B.1: Relations .....

- ☐ **Verify that a set is a relation from set A to set B.**
- ☐ **Represent a relation using suitable format (3 options)**
- ☐ **Verify that a relation has/does not have various properties**

B.2: Functions .....

- ☐ **Represent a function using suitable format (6 options)**
- ☐ **Verify that a function has/does not have various properties in particular injective, surjective and bijective**
- ☐