

# Discrete Mathematics — Tutorial Sheet 06 — Enumeration

BSc (H) in App Comp, Ent Sys, Comp Foren, and the IoT

## Basic Counting Principles

### Question 1

Your wardrobe consists of 5 shirts, 3 pairs of pants, and 17 bow ties. How many different outfits can you make?

### Question 2

For your college interview, you must wear a tie. You own 3 regular (boring) ties and 5 (cool) bow ties.

- (a) How many choices do you have for your neck-wear?
- (b) You realise that the interview is for clown college, so you should probably wear both a regular tie and a bow tie. How many choices do you have now?
- (c) For the rest of your outfit, you have 5 shirts, 4 skirts, 3 pants, and 7 dresses. You want to select either a shirt to wear with a skirt or pants, or just a dress. How many outfits do you have to choose from?

### Question 3

If  $|A| = 8$  and  $|B| = 5$ , what is  $|A \cup B| + |A \cap B|$ ?

### Question 4

Consider all 5 letter “words” made from the letters  $a$  through  $h$ . (Recall, words are just strings of letters, not necessarily actual English words.)

- (a) How many of these words are there total?
- (b) How many of these words contain no repeated letters?
- (c) How many of these words start with the sub-word “aha”?
- (d) How many of these words either start with “aha” or end with “bah” or both?
- (e) How many of the words containing no repeats also do not contain the sub-word “bad”?

## Binomial Coefficients

### Question 5

Let  $A = \{1, 2, 3, \dots, 9\}$ .

- (a) How many subsets of  $A$  are there? That is, find  $|\mathcal{P}(A)|$ . Explain.
- (b) How many subsets of  $A$  contain exactly 5 elements? Explain.
- (c) How many subsets of  $A$  contain only even numbers? Explain.
- (d) How many subsets of  $A$  contain an even number of elements? Explain.

**Question 6**

How many 9-bit strings (that is, bit strings of length 9) are there which satisfy each of the following criteria? Explain your answers.

- (a) Start with the sub-string 101.
- (b) Have weight 5 (i.e., contain exactly five 1's) and start with the sub-string 101.
- (c) Either start with 101 or end with 11 (or both).
- (d) Have weight 5 and either start with 101 or end with 11 (or both).

**Question 7**

How many shortest lattice paths start at (3,3) and

- (a) end at (10,10)?
- (b) end at (10,10) and pass through (5,7)?
- (c) end at (10,10) and avoid (5,7)?

**Question 8**

What is the coefficient of  $x^{12}$  in  $(x + 2)^{15}$ ?

Combinations and Permutations

**Question 9**

A pizza parlour offers 10 toppings.

- (a) How many 3-topping pizzas could they put on their menu? Assume double toppings are not allowed.
- (b) How many total pizzas are possible, with between zero and ten toppings (but not double toppings) allowed?
- (c) The pizza parlour will list the 10 toppings in two equal-sized columns on their menu. How many ways can they arrange the toppings in the left column?

**Question 10**

Using the digits 2 through 8, find the number of different 5-digit numbers such that:

- (a) Digits can be used more than once.
- (b) Digits cannot be repeated, but can come in any order.
- (c) Digits cannot be repeated and must be written in increasing order.
- (d) Which of the above counting questions is a combination and which is a permutation? Explain why this makes sense.

**Question 11**

How many triangles are there with vertices from the points shown below? Note, we are not allowing degenerate triangles — ones with all three vertices on the same line, but we do allow non-right triangles. Explain why your answer is correct.



**Hint.** You need exactly two points on either the  $x$ - or  $y$ -axis, but don't over-count the right triangles.

Advanced Counting Techniques

**Question 12**

After gym class you are tasked with putting the 14 identical dodgeballs away into 5 bins.

- (a) How many ways can you do this if there are no restrictions?
- (b) How many ways can you do this if each bin must contain at least one dodgeball?

**Question 13**

How many integer solutions are there to the equation  $x + y + z = 8$  for which

- (a)  $x$ ,  $y$ , and  $z$  are all positive?
- (b)  $x$ ,  $y$ , and  $z$  are all non-negative?
- (c)  $x$ ,  $y$ , and  $z$  are all greater than  $-3$ .

**Question 14**

Using the digits 2 through 8, find the number of different 5-digit numbers such that:

- (a) Digits cannot be repeated and must be written in increasing order. For example, 23678 is okay, but 32678 is not.
- (b) Digits *can* be repeated and must be written in *non-decreasing* order. For example, 24448 is okay, but 24484 is not.